Challenger-03

1. Prove that $\sin 10^\circ \sin 30^\circ \sin 50^\circ \sin 70^\circ = \frac{1}{16}$

Method 1 $\sin(90-\theta) = \cos\theta$ $\therefore$ $\sin 70^\circ = \cos 20^\circ$
$\sin 50^\circ = \cos 40^\circ$ etc.

we know

$$\cos\theta \cos 2\theta \cos 4\theta \ldots \cos 2^{n-1}\theta = \frac{1}{2^n}\frac{\sin 2^n\theta}{\sin\theta}$$

$\therefore$ Above expression will be

$$= \frac{1}{2}\sin 70^\circ \sin 50^\circ \sin 10^\circ \qquad \left[\because \sin 30^\circ = \frac{1}{2}\right]$$

$$= \frac{1}{2}\cos 20^\circ \cos 40^\circ \cos 80^\circ = \frac{1}{2}\times\frac{1}{2^3}\frac{\sin 160}{\sin 20}$$

$$= \frac{1}{16}\frac{\sin(180-20)}{\sin 20} = \frac{1}{16}\frac{\sin 20}{\sin 20} = \frac{1}{16} \text{ Ans.}$$

Method 2. $\sin 10^\circ \sin 30^\circ \sin 50^\circ \sin 70^\circ$

$$= \frac{1}{2\times 2}2\sin 50^\circ \sin 10^\circ \sin 70^\circ \qquad [\because 2\sin A\sin B = \cos(A-B) - \cos(A+B)]$$

$$= \frac{1}{4}\left[\cos 40^\circ - \cos 60^\circ\right]\sin 70^\circ$$

$$= \frac{1}{4}\left[\cos 40 - \frac{1}{2}\right]\sin 70 = \frac{1}{4}\left[\sin 70\cos 40 - \frac{1}{2}\sin 70\right]$$

$$= \frac{1}{4}\left[\frac{1}{2}2\sin 70\cos 40 - \frac{1}{2}\sin 70\right] \qquad [\because 2\sin A\cos B = \sin(A+B)+\sin(A-B)]$$

$$= \frac{1}{4}\left[\frac{1}{2}(\sin 110^\circ + \sin 30^\circ) - \frac{1}{2}\sin 70^\circ\right] \qquad \because \sin 110^\circ = \sin(180-70) = \sin 70^\circ$$

$$= \frac{1}{4}\left[\cancel{\frac{1}{2}\sin 110^\circ} + \frac{1}{4} - \cancel{\frac{1}{2}\sin 70}\right] = \frac{1}{16}$$

2. Prove that $\tan 20^\circ \tan 40^\circ \tan 80^\circ = \tan 60^\circ$

Soln. $\tan 80^\circ \tan 40^\circ \tan 20^\circ$

$$= \frac{\sin 80^\circ \sin 40 \sin 20}{\cos 80 \cos 40 \cos 20} = \frac{2\sin 80\sin 40\sin 20}{2\cos 80\cos 40\cos 20}$$

$$= \frac{[\cos(80-40) - \cos(80+40)]\sin 20}{[\cos(80+40) + \cos(80-40)]\cos 20}$$

$$= \frac{(\cos 40 - \cos 120)}{(\cos 120 + \cos 40)} \frac{\sin 20}{\cos 20}$$

$$= \frac{2 \cos 40 \sin 20 - 2 \cos 120 \sin 20}{2 \cos 120 \cos 20 + 2 \cos 40 \cos 20}$$

$$= \frac{\sin 60 - \sin 20 - 2\left(-\frac{1}{2}\right) \sin 20}{2\left(-\frac{1}{2}\right) \cos 20 + \cos 60 + \cos 20}$$

$$= \frac{\frac{\sqrt{3}}{2} - \cancel{\sin 20} + \cancel{\sin 20}}{-\cancel{\cos 20} + \frac{1}{2} + \cancel{\cos 20}} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$$

$$= \tan 60^\circ$$

Prove that

3. (i) $(\sin 3A + \sin A) \sin A + (\cos 3A - \cos A) \cos A = 0$

Soln

$$\sin 3A \sin A + \sin^2 A + \cos 3A \cos A - \cos^2 A$$

$$= \cos 3A \cos A + \sin 3A \cos A - (\cos^2 A - \sin^2 A)$$

$$= \cos(3A - A) - \cos 2A = \cos 2A - \cos 2A = 0$$

(ii) $\cos 2\theta \cos \frac{\theta}{2} - \cos 3\theta \cos \frac{9\theta}{2} = \sin 5\theta \sin \frac{5\theta}{2}$

Soln

$$\frac{1}{2}\left[2 \cos 2\theta \cos \frac{\theta}{2} - 2 \cos \frac{9\theta}{2} \cos 3\theta\right]$$

$$= \frac{1}{2}\left[\left(\cos \frac{5\theta}{2} + \cancel{\cos \frac{3\theta}{2}}\right) - \left(\cos \frac{15\theta}{2} + \cancel{\cos \frac{3\theta}{2}}\right)\right]$$

$$= \frac{1}{2}\left[\cos \frac{5\theta}{2} - \cos \frac{15\theta}{2}\right] = \frac{1}{\cancel{2}} \cancel{2} \sin 5\theta \sin \frac{5\theta}{2}$$

$$= \sin 5\theta \sin \frac{5\theta}{2}$$

(iii) $\sin\alpha + \sin\left(\alpha + \frac{2\pi}{3}\right) + \sin\left(\alpha + \frac{4\pi}{3}\right) = 0$

Soln $\sin\alpha + 2\sin\left(\frac{\alpha + \frac{2\pi}{3} + \alpha + \frac{4\pi}{3}}{2}\right)\cos\left(\frac{\cancel{\alpha} + \frac{4\pi}{3} - \cancel{\alpha} - \frac{2\pi}{3}}{2}\right)$

$$= \sin\alpha + 2\sin(\alpha + \pi)\cos\frac{\pi}{3}$$

$$= \sin\alpha - \cancel{2}\sin\alpha\left(\frac{1}{\cancel{2}}\right) = \sin\alpha - \sin\alpha = 0$$

4. Prove that

$$\frac{\cos 4x + \cos 3x + \cos 2x}{\sin 4x + \sin 3x + \sin 2x}$$

Soln $\dfrac{\cos 4x + \cos 2x + \cos 3x}{\sin 4x + \sin 2x + \sin 3x}$

$$= \frac{2\cos 3x \cos x + \cos 3x}{2\sin 3x \cos x + \sin 3x}$$

$$= \frac{\cos 3x\,\cancel{(2\cos x + 1)}}{\sin 3x\,\cancel{(2\cos x + 1)}} = \cot 3x$$

5. Prove that

$$\frac{\sin A + \sin 3A + \sin 5A + \sin 7A}{\cos A + \cos 3A + \cos 5A + \cos 7A} = \tan 4A$$

Soln $\dfrac{\sin 7A + \sin A + \sin 5A + \sin 3A}{\cos 7A + \cos A + \cos 5A + \cos 3A}$

$$= \frac{2\sin 4A \cos 3A + 2\sin 4A \cos A}{2\cos 4A \cos 3A + 2\cos 4A \cos A}$$

$$= \frac{\cancel{2}\sin 4A\,\cancel{(\cos 3A + \cos A)}}{\cancel{2}\cos 4A\,\cancel{(\cos 3A + \cos A)}} = \tan 4A$$

6. Prove that

$$\frac{\cos 2A \cos 3A - \cos 2A \cos 7A + \cos A \cos 10A}{\sin 4A \sin 3A - \sin 2A \sin 5A + \sin 7A \sin 4A} = \cot 6A \cot 5A$$

Soln:

$$\frac{\cos 2A(\cos 3A - \cos 7A) + \cos 10A \cos A}{\sin 4A(\sin 3A + \sin 7A) - \sin 2A \sin 5A}$$

$$= \frac{\cos 2A\, 2\sin 5A \sin 2A + \cos 10A \cos A}{\sin 4A\, 2\sin 5A \cos 2A - \sin 2A \sin 5A}$$

$$= \frac{\sin 5A\, 2\sin 2A \cos 2A + \cos 10A \cos A}{\sin 5A\,(2\sin 4A \cos 2A - \sin 2A)}$$

$$= \frac{2\sin 5A \sin 4A + 2\cos 10A \cos A}{2\sin 5A\,(\sin 6A + \cancel{\sin 2A} - \cancel{\sin 2A})}$$

$$= \frac{\cos A - \cancel{\cos 9A} + \cos 11A + \cancel{\cos 9A}}{2\sin 6A \sin 5A}$$

$$= \frac{\cancel{2}\cos 6A \cos 5A}{\cancel{2}\sin 6A \sin 5A} = \cot 6A \cot 5A$$

(7) $\left(\frac{\cos A + \cos B}{\sin A - \sin B}\right)^n + \left(\frac{\sin A + \sin B}{\cos A - \cos B}\right)^n$

$$= \begin{cases} 2\cot^n\left(\frac{A-B}{2}\right), & \text{if } n \text{ is even} \\ 0, & \text{if } n \text{ is odd.} \end{cases}$$

Soln. $\left(\frac{\cancel{2\cos\frac{A+B}{2}}\cos\frac{A-B}{2}}{\cancel{2\cos\frac{A+B}{2}}\sin\frac{A-B}{2}}\right)^n + \left(\frac{\cancel{2\sin\frac{A+B}{2}}\cos\frac{A-B}{2}}{-\cancel{2\sin\frac{A+B}{2}}\sin\frac{A-B}{2}}\right)^n$

$$= \left(\cot\frac{A-B}{2}\right)^n + \left(-\cot\frac{A-B}{2}\right)^n$$

$$= \cot^n\left(\frac{A-B}{2}\right) + (-1)^n\cot^n\left(\frac{A-B}{2}\right)$$

$= 2\cot^n\left(\frac{A-B}{2}\right)$ when n is even.

as $(-1)^n = 1$

$= 0$ when n is an odd.

$(-1)^n = -1$

8. If $\frac{\sin(\theta+\alpha)}{\cos(\theta-\alpha)} = \frac{1-m}{1+m}$, prove that

$$\tan\left(\frac{\pi}{4}-\theta\right)\tan\left(\frac{\pi}{4}-\alpha\right) = m$$

Soln: using componendo and dividendo.

$$\frac{(\cancel{1}+m)-(\cancel{1}-m)}{(1+\cancel{m})+(1-\cancel{m})} = \frac{\cos(\theta-\alpha) - \sin(\theta+\alpha)}{\cos(\theta-\alpha) + \sin(\theta+\alpha)}$$

$$\Rightarrow \frac{\cancel{2}m}{\cancel{2}} = \frac{\sin\left[\frac{\pi}{2}-(\theta-\alpha)\right] - \sin(\theta+\alpha)}{\sin\left[\frac{\pi}{2}-(\theta-\alpha)\right] + \sin(\theta+\alpha)}$$

$$\Rightarrow m = \frac{\cancel{2}\cos\left[\frac{\frac{\pi}{2}-\cancel{\theta}+\alpha+\cancel{\theta}+\alpha}{2}\right)\sin\left(\frac{\frac{\pi}{2}-\theta+\cancel{\alpha}-\theta-\cancel{\alpha}}{2}\right)}{\cancel{2}\sin\left[\frac{\frac{\pi}{2}-\cancel{\theta}+\alpha+\cancel{\theta}+\alpha}{2}\right]\cos\left(\frac{\frac{\pi}{2}-\theta+\cancel{\alpha}-\theta-\cancel{\alpha}}{2}\right)}$$

$$\Rightarrow m = \frac{\cos\left(\frac{\pi}{4}+\alpha\right)\sin\left(\frac{\pi}{4}-\theta\right)}{\sin\left[\frac{\pi}{4}+\alpha\right]\cos\left[\frac{\pi}{4}-\theta\right]}$$

$$\Rightarrow m = \cot\left(\frac{\pi}{4}+\alpha\right)\tan\left(\frac{\pi}{4}-\theta\right)$$

$$= \tan\left[\frac{\pi}{2}-\left(\frac{\pi}{4}+\alpha\right)\right]\tan\left(\frac{\pi}{4}-\theta\right)$$

$$= \tan\left(\frac{\pi}{4}-\alpha\right)\tan\left(\frac{\pi}{4}-\theta\right)$$ Proved.

9. If $\sin\theta = n\sin(\theta+2\alpha)$ prove that

$$\tan(\theta+\alpha) = \frac{1+n}{1-n}\tan\alpha$$

Soln: $\frac{1}{n} = \frac{\sin(\theta+2\alpha)}{\sin\theta}$

using componendo and dividendo we get

$$\frac{1+n}{1-n} = \frac{\sin(\theta+2\alpha)+\sin\theta}{\sin(\theta+2\alpha)-\sin\theta}$$

$$= \frac{\cancel{2}\sin(\theta+\alpha)\cos\alpha}{\cancel{2}\cos(\theta+\alpha)\sin\alpha}$$

$$= \frac{\tan(\theta+\alpha)}{\tan\alpha}$$

$$\therefore \left(\frac{1+n}{1-n}\right)\tan\alpha = \tan(\theta+\alpha)$$

10. If $a\sin\theta = b\sin\left(\theta+\frac{2\pi}{3}\right) = c\sin\left(\theta+\frac{4\pi}{3}\right)$

prove that $ab+bc+ca=0$

Soln: $a\sin\theta = b\sin\left(\theta+\frac{2\pi}{3}\right)$

$$\Rightarrow \frac{a}{b} = \frac{\sin\left(\theta+\frac{2\pi}{3}\right)}{\sin\theta} = \frac{\sin\theta\cos\frac{2\pi}{3}+\cos\theta\sin\frac{2\pi}{3}}{\sin\theta}$$

$$= \cos\frac{2\pi}{3} + \cot\theta\sin\frac{2\pi}{3}$$

$$= -\frac{1}{2} + \frac{\sqrt{3}}{2}\cot\theta \quad \text{——— (1)}$$

Also,

$$\frac{a}{c} = \frac{\sin\left(\theta + \frac{4\pi}{3}\right)}{\sin\theta}$$

$$= \frac{\sin\theta \cos\frac{4\pi}{3} + \cos\theta \sin\frac{4\pi}{3}}{\sin\theta}$$

$$= \cos\frac{4\pi}{3} + \cot\theta \sin\frac{4\pi}{3}$$

$$= -\frac{1}{2} - \frac{\sqrt{3}}{2}\cot\theta \quad \text{—(2)}$$

Adding (1) and (2), we get

$$\frac{a}{b} + \frac{a}{c} = -1$$

$$ac + ab = -bc$$

$$ab + bc + ac = 0$$

11. If $\alpha, \beta, \gamma \in \left(0, \frac{\pi}{2}\right)$, prove that

$$\sin\alpha + \sin\beta + \sin\gamma > \sin(\alpha+\beta+\gamma)$$

Soln. $\because$ α, β, γ are all acute angles

Now, we can show it as

$$\sin\alpha + \sin\beta + \sin\gamma - \sin(\alpha+\beta+\gamma) > 0$$

$$= 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} + 2\cos\left(\frac{\gamma+\alpha+\beta+\gamma}{2}\right)\sin\left\{-\left(\frac{\alpha+\beta}{2}\right)\right\}$$

$$= 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} - 2\cos\left(\frac{\alpha+\beta+2\gamma}{2}\right)\sin\left(\frac{\alpha+\beta}{2}\right)$$

$$= 2\sin\frac{\alpha+\beta}{2}\left[\cos\frac{\alpha-\beta}{2} - \cos\frac{\alpha+\beta+2\gamma}{2}\right]$$

$$= 2\sin\frac{\alpha+\beta}{2}\, 2\sin\left(\frac{\frac{\alpha-\cancel{\beta}}{2}+\frac{\alpha+\cancel{\beta}+2\gamma}{2}}{2}\right)\sin\left(\frac{\frac{\cancel{\alpha}+\beta+2\gamma}{2}-\frac{\cancel{\alpha}-\beta}{2}}{2}\right)$$

$$= 4\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha+\gamma}{2}\right)\sin\left(\frac{\beta+\gamma}{2}\right) > 0$$

$\because \frac{\alpha+\beta}{2} \in \left(0, \frac{\pi}{2}\right)$ as $\alpha, \beta, \gamma \in (0, \pi/2)$

$\therefore \sin\left(\frac{\alpha+\beta}{2}\right)$ will be positive as it lies in 1st quadrant.

12. If $\frac{\tan(\theta+\alpha)}{a} = \frac{\tan(\theta+\beta)}{b} = \frac{\tan(\theta+\gamma)}{c}$

prove that

$$\frac{a+b}{a-b}\sin^2(\alpha-\beta) + \frac{b+c}{b-c}\sin^2(\beta-\gamma) + \frac{c+a}{c-a}\sin^2(\gamma-\alpha) = 0$$

Sol^n considering, $\frac{\tan(\theta+\alpha)}{a} = \frac{\tan(\theta+\beta)}{b}$

$$\Rightarrow \frac{a}{b} = \frac{\tan(\theta+\alpha)}{\tan(\theta+\beta)} \Rightarrow \frac{a+b}{a-b} = \frac{\tan(\theta+\alpha)+\tan(\theta+\beta)}{\tan(\theta+\alpha)-\tan(\theta+\beta)}$$

$$\Rightarrow \frac{a+b}{a-b} = \frac{\dfrac{\sin(\theta+\alpha)}{\cos(\theta+\alpha)} + \dfrac{\sin(\theta+\beta)}{\cos(\theta+\beta)}}{\dfrac{\sin(\theta+\alpha)}{\cos(\theta+\alpha)} - \dfrac{\sin(\theta+\beta)}{\cos(\theta+\beta)}}$$

$$= \frac{\sin(\theta+\alpha)\cos(\theta+\beta) + \cos(\theta+\alpha)\sin(\theta+\beta)}{\sin(\theta+\alpha)\cos(\theta+\beta) - \cos(\theta+\alpha)\sin(\theta+\beta)}$$

$$= \frac{\sin(\theta+\alpha+\theta+\beta)}{\sin(\cancel{\theta}+\alpha-\cancel{\theta}-\beta)}$$

$$\left(\frac{a+b}{a-b}\right)\sin(\alpha-\beta) = \sin(2\theta+\alpha+\beta)$$

$$\left(\frac{a+b}{a-b}\right)\sin^2(\alpha-\beta) = \frac{1}{2}\, 2\sin(2\theta+\alpha+\beta)\sin(\alpha-\beta)$$

$$= \frac{1}{2}\left[\cos(2\theta+\cancel{\alpha}+\beta-\cancel{\alpha}+\beta) - \cos(2\theta+\alpha+\cancel{\beta}+\alpha-\cancel{\beta})\right]$$

$$= \frac{1}{2}\left[\cos 2(\theta+\beta) - \cos 2(\theta+\alpha)\right]$$

$$= \frac{1}{2}\left[\{\cancel{1}-2\sin^2(\theta+\beta)\} - \{\cancel{1}-2\sin^2(\theta+\alpha)\}\right]$$

$$\frac{a+b}{a-b}\sin^2(\alpha-\beta) = \sin^2(\theta+\alpha) - \sin^2(\theta+\beta)$$

Similarly, $\dfrac{b+c}{b-c}\sin^2(\beta-\gamma) = \sin^2(\theta+\beta) - \sin^2(\theta+\gamma)$

$$\frac{c+a}{c-a}\sin^2(\gamma-\alpha) = \sin^2(\theta+\gamma) - \sin^2(\theta+\alpha)$$

Adding all we get,

$$\frac{a+b}{a-b}\sin^2(\alpha-\beta) + \frac{b+c}{b-c}\sin^2(\beta-\gamma) + \frac{c+a}{c-a}\sin^2(\gamma-\alpha) = 0$$

13. $\dfrac{\sin x}{\cos 3x} + \dfrac{\sin 3x}{\cos 9x} + \dfrac{\sin 9x}{\cos 27x} = \dfrac{1}{2}(\tan 27x - \tan x)$

Soln We have

$$\frac{\sin x}{\cos 3x} = \frac{2\sin x\cos x}{2\cos x\cos 3x} = \frac{\sin 2x}{2\cos 3x\cos x}$$

$$= \frac{\sin(3x-x)}{2\cos 3x\cos x}$$

$$= \frac{1}{2}\left[\frac{\sin 3x\cos x - \cos 3x\sin x}{\cos 3x\cos x}\right]$$

$$\frac{\sin x}{\cos 3x} = \frac{1}{2}\left[\tan 3x - \tan x\right] \quad \text{—(I)}$$

Similarly,

$$\frac{\sin 3x}{\cos 9x} = \frac{1}{2}\left[\tan 9x - \tan 3x\right] \quad \text{—(II)}$$

$$\frac{\sin 9x}{\cos 27x} = \frac{1}{2}\left[\tan 27x - \tan 9x\right] \quad \text{—(III)}$$

Adding (i), (ii) and (iii) we get.

$$\frac{\sin x}{\cos 3x} + \frac{\sin 3x}{\cos 9x} + \frac{\sin 9x}{\cos 27x} = \frac{1}{2}\left[\tan 27x - \tan x\right]$$

14. Prove that

$$\tan\theta \tan\left(\theta - \frac{\pi}{3}\right) + \tan\theta \tan\left(\theta + \frac{\pi}{3}\right) + \tan\left(\theta - \frac{\pi}{3}\right)\tan\left(\theta + \frac{\pi}{3}\right) = -3$$

Sol'n we can write the above as

$$\left[1 + \tan\theta \tan\left(\theta - \frac{\pi}{3}\right)\right] + \left[1 + \tan\theta \tan\left(\theta + \frac{\pi}{3}\right)\right] + \left[1 + \tan\left(\theta + \frac{\pi}{3}\right)\tan\left(\theta - \frac{\pi}{3}\right)\right] = 0$$

we know

$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$$\therefore \quad 1 + \tan A \tan B = \tan(A - B)\left[\tan A - \tan B\right]$$

$\therefore$ ~~Applying~~ the above formula. we have,

$$\tan\frac{\pi}{3}\left[\tan\theta - \tan\left(\theta - \frac{\pi}{3}\right)\right] + \tan\frac{\pi}{3}\left[\tan\left(\theta + \frac{\pi}{3}\right) - \tan\theta\right] + \tan\frac{2\pi}{3}\left[\tan\left(\theta + \frac{\pi}{3}\right) - \tan\left(\theta - \frac{\pi}{3}\right)\right]$$

$$= \sqrt{3}\left[\cancel{\tan\theta} - \tan\left(\theta - \frac{\pi}{3}\right) + \tan\left(\theta + \frac{\pi}{3}\right) - \cancel{\tan\theta}\right] - \sqrt{3}\left[\tan\left(\theta + \frac{\pi}{3}\right) - \tan\left(\theta - \pi/3\right)\right]$$

$$= \sqrt{3}\left[\cancel{\tan\left(\theta+\frac{\pi}{3}\right)} - \cancel{\tan\left(\theta-\frac{\pi}{3}\right)} - \cancel{\tan\left(\theta+\frac{\pi}{3}\right)} + \cancel{\tan\left(\theta-\frac{\pi}{3}\right)}\right]$$

$$= \sqrt{3} \times 0 = 0 \text{ Proved.}$$

15. Prove that

$$\cot 16^\circ \cot 44^\circ + \cot 44^\circ \cot 76^\circ - \cot 76^\circ \cot 16^\circ = 3$$

Soln.

We have $\cot(A+B) = \dfrac{\cot A \cot B - 1}{\cot B + \cot A}$

$\therefore \cot A \cot B - 1 = \cot(A+B)\left[\cot B + \cot A\right]$

L.H.S.

$$\therefore (\cot 44 \cot 16 - 1) + (\cot 76 \cot 44 - 1) - (\cot 76 \cot 16 + 1) + 3$$

$$= \cot 60\left[\cot 44^\circ + \cot 16^\circ\right] + \cot 120\left[\cot 76^\circ + \cot 44^\circ\right] - \cot 60\left[\cot 16^\circ - \cot 76^\circ\right] + 3$$

$$= \frac{1}{\sqrt{3}}\left[\cot 44^\circ + \cot 16^\circ\right] - \frac{1}{\sqrt{3}}\left[\cot 76^\circ + \cot 44^\circ\right] - \frac{1}{\sqrt{3}}\left[\cot 16^\circ - \cot 76^\circ\right] + 3$$

$$= 3 + \frac{1}{\sqrt{3}}\left[\cancel{\cot 44^\circ} + \cancel{\cot 16^\circ} - \cancel{\cot 76^\circ} - \cancel{\cot 44^\circ} - \cancel{\cot 16^\circ} + \cancel{\cot 76^\circ}\right]$$

$$= 3 + \frac{1}{\sqrt{3}} \times 0 = 3 \text{ Proved.}$$

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