Q.1 The domain of the function

$$f(x) = \log_{10}\left(\sqrt{x-4} + \sqrt{6-x}\right)$$

Soln:

$$x - 4 \geq 0 \Rightarrow x \geq 4 \quad \text{— (i)}$$

$$6 - x \geq 0 \Rightarrow x \leq 6 \quad \text{— (ii)}$$

When $\leq x \leq 6$

Then $\sqrt{x-4} + \sqrt{6-x} > 0$

$\therefore$ domain $x \in [4, 6]$

Q.2 Range of the function defined by $f(x) = \left[\dfrac{1}{\sin\{x\}}\right]$ where $[x]$ and $\{x\}$ denotes respectively greatest integer and fractional part of x respectively.

Soln:

$$0 \leq \{x\} < 1$$

$$\therefore\ 0 \leq \sin\{x\} < \sin 1 < 1$$

$$\therefore\ 1 < \frac{1}{\sin\{x\}} < \infty$$

$$\therefore\ \left[\frac{1}{\sin\{x\}}\right] = 1, 2, 3, \ldots$$

i.e a set of natural Numbers.

Q.3 The range of function

$$y = \frac{x}{1+x^2}$$

Soln: $y + yx^2 = x$

$\Rightarrow x^2 y - x + y = 0$

For real x, $D \geq 0$

$\therefore (-1)^2 - 4 \times y \times y \geq 0$

$\Rightarrow 4y^2 - 1 \leq 0$

$\Rightarrow y^2 - \frac{1}{4} \leq 0$

$\Rightarrow (y - \frac{1}{2})(y + \frac{1}{2}) \leq 0$

$\Rightarrow -\frac{1}{2} \leq y \leq \frac{1}{2}$

$\therefore$ Range $[-\frac{1}{2}, \frac{1}{2}]$

Q.4 The range of the function

$$y = \frac{x^2}{1+x^2}$$

Soln: $y + yx^2 = x^2$

$\Rightarrow (1-y)x^2 = y$

$\therefore x^2 = \frac{y}{1-y}$

$\because x^2 \geq 0$

$\Rightarrow \frac{y}{1-y} \geq 0 \Rightarrow \frac{y}{y-1} \leq 0$

$0 \leq y \leq 1$ $\therefore$ Range $[0, 1]$

5. The range of the function

$$y = \frac{1}{2-\sin 3x}$$

Soln.

$\because -1 \le \sin 3x \le 1$

$\therefore -1 \le -\sin 3x \le 1$

$\Rightarrow 1 \le 2-\sin 3x \le 3$.

$\Rightarrow \frac{1}{3} \le \frac{1}{2-\sin 3x} \le 1$

$\therefore$ Range $\left[\frac{1}{3}, 1\right]$

6. The range of the function

$$f(x) = \log_e(3x^2-4x+5)$$

Soln let $y = \log_e(3x^2-4x+5)$

$\therefore 3x^2-4x+5 = e^y$

$\Rightarrow 3x^2-4x+5-e^y = 0$

for real x, $D \ge 0$

$\therefore (-4)^2 - 4\times 3\times(5-e^y) \ge 0$

$16 - 60 + 12e^y \ge 0$

$\Rightarrow 12e^y \ge 44$

$\Rightarrow e^y \ge \frac{11}{3}$

$\Rightarrow y \ge \log_e \frac{11}{3}$

$\therefore$ Range $\left[\log_e \frac{11}{3}, \infty\right)$

7 The range of the function

$$y = 3\sin\sqrt{\frac{\pi^2}{16} - x^2}$$

Soln For domain $\frac{\pi^2}{16} - x^2 \geq 0$

$\Rightarrow \quad -\frac{\pi}{4} \leq x \leq \frac{\pi}{4}$

$\therefore$ when $x = \frac{\pi}{4}$

$y = 3\sin 0 = 0$

when $x = 0$

$y = 3\sin\sqrt{\frac{\pi^2}{16} - 0} = 3\sin\frac{\pi}{4}$

$= \frac{3}{\sqrt{2}}$

$\therefore$ Range $\left[0, \frac{3}{\sqrt{2}}\right]$

8. The range of function

$$f(x) = \sqrt{3x^2 - 4x + 5}$$

Soln: $3x^2 - 4x + 5 = 3\left(x^2 - \frac{4}{3}x + \frac{5}{3}\right)$

$= 3\left(x - \frac{2}{3}\right)^2 + \frac{11}{9} > 0 \ \forall\, x \in R$

$\therefore$ minimum value $f(x) = \frac{\sqrt{11}}{3}$

when $x = 2/3$

maximum value of $f(x) = \infty$

$\therefore$ Range $\left[\frac{\sqrt{11}}{3}, \infty\right)$

9. The value of function

$$f(x) = \frac{x^2 - 3x + 2}{x^2 + x - 6}$$ lies in

Soln: $$y = \frac{x^2 - 3x + 2}{x^2 + x - 6}$$

$\Rightarrow (y-1).x^2 + (y+3)x - 2(3y+1) = 0$

$\therefore$ for real x, $1 \geq 0$

$\Rightarrow (y+3)^2 + 4(y-1)2(3y+1) \geq 0$

$\Rightarrow y^2 + 6y + 9 + 8(3y^2 - 2y - 1) \geq 0$

$\Rightarrow 25y^2 - 10y + 1 \geq 0$

$\Rightarrow (5y-1)^2 \geq 0 \quad \forall\, y \in R$

$\therefore$ Range $(-\infty, \infty)$

10. The range of function

$$f(x) = \tan\sqrt{\frac{\pi^2}{9} - x^2}$$

Soln: $\frac{\pi^2}{9} - x^2 \geq 0 \Rightarrow -\frac{\pi}{3} \leq x \leq \frac{\pi}{3}$

when $x = \pi/3$, $f(x) = \tan 0 = 0$

$x = 0$, $f(x) = \tan \pi/3 = \sqrt{3}$

$\therefore$ Range $[0, \sqrt{3})$

11. The range of function

$$f(x) = \sin x - \cos x$$

Soln $-\sqrt{a^2+b^2} \leq a \sin\theta \pm b\cos\theta \leq \sqrt{a^2+b^2}$

$\therefore -\sqrt{1^2+1^2} \leq \sin x - \cos x \leq \sqrt{1^2+1^2}$

$\therefore$ Range $[-\sqrt{2}, \sqrt{2}]$

Q.12 The Range of Function

$$f(x) = \frac{x}{|x|}, \quad x \neq 0$$

Soln: when $x > 0$, $f(x) = \frac{x}{x} = 1$

when $x < 0$, $f(x) = \frac{x}{-x} = -1$

$\therefore$ Range $\{-1, 1\}$

Q.13 The Period of the function

$$f(x) = a\sin kx + b\cos kx$$

Soln: Period of $\sin kx$ is $\frac{2\pi}{k}$

Period of $\cos kx$ is $\frac{2\pi}{k}$

$\therefore$ Period of $a\sin kx + b\cos kx$

is Lcm of $\frac{2\pi}{k}$ and $\frac{2\pi}{k}$ i.e $\frac{2\pi}{k}$

but Period is a Positive value

$\therefore$ Period would be $\frac{2\pi}{|k|}$

Q.14 Period of $f(x) = \cos x^2$

Soln: $f(x) = \cos x^2$ is not a Periodic function

Q.15 The period of $f(x) = \sin\sqrt{x}$

Soln $f(x) = \sin\sqrt{x}$ is not a periodic function

Q.16. The Period of $f(x) = \sin^4 2x + \cos^4 2x$

Soln: $\because$ Period of $\sin^4 x + \cos^4 x$ is $\pi/2$

$\therefore$ Period of $\sin^4 2x + \cos^4 2x$ is $\pi/4$

Q.17. The Period of function

$$f(x) = \cos\left(\frac{8x+5}{4\pi}\right)$$

Soln. Period of $\cos x$ is 2π

$\therefore$ Period of $f(x) = \cos\left(\frac{2}{\pi}x + \frac{5}{4\pi}\right)$

is $\frac{2\pi}{\frac{2}{\pi}} = \pi^2$

Q.18. The Period of $f(x) = x - [x]$ where $[x]$ denotes the greatest integer function

Soln: $f(x) = x - [x]$

$= \{x\}$ fractional part of x. Period is 1.

Q.19. If T_1 is the Period of the function $y = e^{3(x-[x])}$ and T_2 is Period of the function $y = e^{3x-[3x]}$, where $[\cdot]$ denotes greatest Integer function then which of the following is true?

(a) $T_1 = T_2$ (b) $T_1 = \frac{T_2}{3}$

(c) $T_1 = 3T_2$ (d) None

Soln: $x - [x] = \{x\}$

$\therefore$ Period of $y = e^{3(x-[x])}$

$= e^{3\{x\}}$ is 1.

$\therefore T_1 = 1$

Also, $3x - [3x] = \{3x\}$

$\therefore$ period of $y = e^{3x-[3x]}$

$= e^{\{3x\}}$ is $\frac{1}{3}$

$\therefore T_2 = \frac{1}{3}$

Hence, $T_1 = 3T_2$

Q.20 The period of

$f(x) = \cos\left(\frac{\pi x}{n!}\right) - \sin\left(\frac{\pi x}{(n+1)!}\right)$

Soln Period is LCM of

$\frac{2\pi}{\frac{\pi}{n!}}$ and $\frac{2\pi}{\frac{\pi}{(n+1)!}}$ is equal to

LCM of $2(n!)$ and $2(n+1)!$

i.e $2(n+1)!$

Q.21. The period of the function

$f(x) = 2\sin x + 3\cos 2x$

Soln Period of $\sin x$ is 2π

Period of $\cos 2x$ is $\frac{2\pi}{2} = \pi$

$\therefore$ period of $2\sin x + 3\cos 2x$ is 2π