Conformal-11

1. If the root of the equations $x^2+px+q=0$ and $x^2+\alpha x+\beta=0$ is common. Then the value of will (where $p \neq \alpha$ and $q \neq \beta$) be

(1) $\dfrac{q-\beta}{\alpha-p}$ (2) $\dfrac{p\beta-\alpha q}{q-\beta}$

(3) $\dfrac{q-\beta}{\alpha-p}$ or $\dfrac{p\beta-\alpha q}{q-\beta}$ (4) None of these

Soln: $\because$ $x^2+px+q=0$ and $x^2+\alpha x+\beta=0$ has a common roots. Let k be a common roots of both of equation

$\therefore$ They must satisfy the equations

$$k^2+pk+q=0$$
$$k^2+\alpha k+\beta=0$$

By cross multiplication,

$$\underset{(\text{i})}{\frac{k^2}{p\beta-q\alpha}} = \underset{(\text{ii})}{\frac{k}{q-\beta}} = \underset{(\text{iii})}{\frac{1}{\alpha-p}}$$

from (i) and (ii) $\quad k=\dfrac{p\beta-q\alpha}{q-\beta}$

from (ii) and (iii) $K = \frac{q-\beta}{\alpha-p}$

Hence, roots are either $\frac{p\beta - q\alpha}{q-\beta}$ or $\frac{q-\beta}{\alpha-p}$

2. If the two equations $x^2 - cx + d = 0$ and $x^2 - ax + b = 0$ have a common root and the second has equal roots, then $2(b+d) =$

(1) 0 (2) $a+c$

(3) ac (4) $-ac$

Soln: Since $x^2 - ax + b = 0$ has equal roots

$\because$ If $ax^2+bx+c=0$ has equal roots, then $D=0$ and roots are $-\frac{b}{2a}$

$\therefore$ roots of equation $x^2 - ax + b = 0$ is $\frac{a}{2}$ and $a^2 = 4b$ —— (i)

Since $\frac{a}{2}$ is common root of both the equation. Hence, it satisfy equation

$$x^2 - cx + d = 0$$

$$\therefore \frac{a^2}{4} - \frac{ac}{2} + d = 0 \Rightarrow a^2 - 2ac + 4d = 0 \quad \text{—— (ii)}$$

from (i) $4b - 2ac + 4d = 0$

$2(b+d) = ac$ Ans.

3. If $x^2 - hx - 21 = 0$, $x^2 - 3hx + 35 = 0$ $(h > 0)$ has a common root, then the value of h is equal to

(1) 1 (2) 2 (3) 3 (4) 4

Soln Since both the equation has a common roots, on solving,

$$x^2 - hx - 21 = x^2 - 3hx + 35$$

$$2hx = 56 \Rightarrow x = \frac{28}{h} \quad \text{— (i)}$$

$\therefore$ from $x^2 - hx - 21 = 0$

$$\frac{(28)^2}{h^2} - 28 - 21 = 0$$

$$\Rightarrow \frac{28 \times 28}{h^2} = 49 \Rightarrow h^2 = \frac{\cancel{28}^{4} \times \cancel{28}^{4}}{\cancel{49}}$$

$\therefore h = 4$ Ans.

4. If every pair of equations $x^2 + px + qr = 0$, $x^2 + qx + rp = 0$, $x^2 + rx + pq = 0$ have a common root, then the sum of three common root is

(1) $-\frac{(p+q+r)}{2}$ (2) $-\frac{p+q+r}{2}$

(3) $-(p+q+r)$ (4) $-p+q+r$

Soln. Let α, β be the roots of equation

$$x^2 + px + qr = 0$$

$$\alpha + \beta = -p \quad \text{—— (i)}$$

β, γ be the roots of equation $x^2 + qx + rp = 0$

$$\therefore \beta + \gamma = -q \quad \text{—— (ii)}$$

and α, γ be the roots of equation $x^2 + rx + pq = 0$

$$\therefore \gamma + \alpha = -r \quad \text{—— (iii)}$$

Adding (i), (ii) and (iii), we get

$$\alpha + \beta + \beta + \gamma + \alpha + \gamma = -p - q - r$$

$$2(\alpha + \beta + \gamma) = -(p + q + r)$$

$$\therefore \alpha + \beta + \gamma = -\left(\frac{p+q+r}{2}\right)$$

ie Sum of three common roots $= \dfrac{-(p+q+r)}{2}$

5. If $ax^2 + bx + c = 0$ and $bx^2 + cx + a = 0$ have a common root $a \neq 0$ Then

$$\frac{a^3 + b^3 + c^3}{abc} =$$

(1) 1 (2) 2

(3) 3 (4) None of these

Soln. Let α be the common root of both the equation, Hence it must satisfy

$$a\alpha^2 + b\alpha + c = 0 \quad \text{— (I)}$$
$$b\alpha^2 + c\alpha + a = 0 \quad \text{— (II)}$$

$$\frac{\alpha^2}{ab-c^2} = \frac{\alpha}{bc-a^2} = \frac{1}{ac-b^2}$$
$$\text{(I)} \qquad \text{(II)} \qquad \text{(III)}$$

$\therefore$ from (II) and (III) $\quad \alpha = \dfrac{bc-a^2}{ac-b^2}$

from (I) and (III) $\quad \alpha^2 = \dfrac{ab-c^2}{ac-b^2}$

$$\therefore \left(\frac{bc-a^2}{ac-b^2}\right)^2 = \frac{ab-c^2}{ac-b^2}$$

$$\Rightarrow (bc-a^2)^2 = (ab-c^2)(ac-b^2)$$

$$\Rightarrow \cancel{b^2c^2} + a^4 - 2a^2bc = a^2bc - ab^3 - ac^3 + \cancel{b^2c^2}$$

$$\Rightarrow a^4 + ab^3 + ac^3 = 3a^2bc$$

$$\Rightarrow (a^3+b^3+c^3)\cancel{a} = (3abc)\cancel{a}$$

$$\Rightarrow \frac{a^3+b^3+c^3}{abc} = 3 \quad \text{Ans.}$$

6. If the equation $x^2+px+q=0$ and $x^2+qx+p=0$ have a common root, then $p+q+1=$

(1) 0 (2) 1 (3) 2 (4) -1

Soln. On solving both the equation

$$\cancel{x^2}+px+q = \cancel{x^2}+qx+p$$

$$(p-q)x-(p-q)=0$$

$$(p-q)(x-1)=0 \Rightarrow x=1$$

$$(\because p\neq q)$$

$\because$ $x=1$ is a common root, Hence it satisfy both the equation

$\therefore$ $x=1$ in $x^2+px+q=0$

$$1+p+q=0 \quad \text{Ans}$$

7. If $x^2+ax+10=0$ and $x^2+bx-10=0$ have a common root, then a^2-b^2 is equal to

(1) 10 (2) 20 (3) 30 (4) 40

Soln. Solving as above

$$\cancel{x^2}+ax+10 = \cancel{x^2}+bx-10$$

$(b-a)x = 20 \quad \therefore x = \frac{20}{b-a}$ — (1)

$\because$ it must satisfy the eqⁿ $x^2 + ax + 10 = 0$

$$\therefore \left(\frac{20}{b-a}\right)^2 + a\left(\frac{20}{b-a}\right) + 10 = 0$$

$$\Rightarrow 400 + 20a(b-a) + 10(b-a)^2 = 0$$

$$\Rightarrow 40 + 2a(b-a) + (b-a)^2 = 0$$

$$\Rightarrow 40 + \cancel{2ab} - 2a^2 + b^2 - \cancel{2ab} + a^2 = 0$$

$$a^2 - b^2 = 40 \quad \underline{\text{Ans}}.$$

8. $x^2 - 11x + a$ and $x^2 - 14x + 2a$ will have a common factor, if $a =$

(1) 24 (2) 0, 24 (3) 3, 24 (4) 0, 3

<u>Solⁿ</u>: If both the eqⁿ has a common factor then both the equation must have a common root.

$\therefore$ Solving both the equation

$$x^2 - 11x + a = x^2 - 14x + 2a$$

$$3x = a \Rightarrow x = \frac{a}{3}$$

which must satisfy the equation

$x^2 - 11x + a = 0$

$\therefore \quad \frac{a^2}{9} - \frac{11a}{3} + a = 0 \Rightarrow a^2 - 33a + 9a = 0$

$\Rightarrow \quad a^2 - 24a = 0 \Rightarrow a(a-24) = 0$

$\Rightarrow a = 0, 24$ Ans.

9. If $x^2 - 3x + 2$ be a factor of $x^4 - px^2 + q$, then $(p, q) =$

(1) $(3, 4)$ (2) $(4, 5)$ (3) $(4, 3)$ (4) $(5, 4)$

Soln. If $x^2 - 3x + 2$ be a factor of $x^4 - px^2 + q$ Then the roots of eqn $x^2 - 3x + 2 = 0$ i.e $(x-2)(x-1) = 0$ ie $x = 1, 2$ must satisfy the equation $x^4 - px^2 + q = 0$

$\therefore \quad (1)^4 - p(1)^2 + q = 0 \Rightarrow p = q + 1$ — (1)

and $(2)^4 - p(2)^2 + q = 0$

$16 - 4p + q = 0$ — (11)

$16 - 4(q+1) + q = 0$

$12 - 3q = 0 \Rightarrow \left.\begin{array}{l} q = 4 \\ p = 5 \end{array}\right\}$ Ans

10. If x is real, then the maximum and minimum values of expression

$$\frac{x^2+14x+9}{x^2+2x+3}$$ will be

(1) 4, -5 (2) 5, -4

(3) -4, 5 (4) -4, -5

Soln: let $y = \dfrac{x^2+14x+9}{x^2+2x+3}$.

$\Rightarrow (y-1)x^2 + 2(y-7)x + 3(y-3) = 0$

for real x, $D \geq 0$

$\Rightarrow [2(y-7)]^2 - 4 \times (y-1) \times 3(y-3) \geq 0$

$\Rightarrow 4(y-7)^2 - 4\times 3(y^2-4y+3) \geq 0$

$\Rightarrow (y^2-14y+49) - 3y^2+12y-9 \geq 0$

$\Rightarrow -2y^2-2y+40 \geq 0$

$\Rightarrow y^2+y-20 \leq 0 \Rightarrow (y+5)(y-4) \leq 0$

$\Rightarrow -5 \leq y \leq 4$

i.e maximum value $= 4$
minimum value $= -5$

11. If x is real, the expression $\dfrac{x+2}{2x^2+3x+6}$ takes all value in the interval

(1) $\left(\frac{1}{13}, \frac{1}{3}\right)$ (2) $\left[-\frac{1}{13}, \frac{1}{3}\right]$

(3) $\left(-\frac{1}{3}, \frac{1}{13}\right)$ (4) None of these

Soln. Let $y = \dfrac{x+2}{2x^2+3x+6}$

$\Rightarrow 2yx^2 + 3yx + 6y = x + 2$

$\Rightarrow 2yx^2 + (3y-1)x + 2(3y-1) = 0$

for real x, $D \geq 0$

$(3y-1)^2 - 4 \times 2y \times 2(3y-1) \geq 0$

$\Rightarrow 9y^2 - 6y + 1 - 48y^2 + 16y \geq 0$

$\Rightarrow 39y^2 - 10y - 1 \leq 0$

$\Rightarrow 39y^2 - 13y + 3y - 1 \leq 0$

$\Rightarrow 13y(3y-1) + 1(3y-1) \leq 0$

$\Rightarrow (3y-1)(13y+1) \leq 0$

$\Rightarrow \left(y - \frac{1}{3}\right)\left(y + \frac{1}{13}\right) \leq 0$

$\therefore -\frac{1}{13} \leq y \leq \frac{1}{3}$ $\therefore$ Ans $\left[-\frac{1}{13}, \frac{1}{3}\right]$

12. If x^2+px+1 is a factor of the expression ax^3+bx+c, then

(1) $a^2+c^2=-ab$ (2) $a^2-c^2=-ab$

(3) $a^2-c^2=ab$ (4) None of these

Soln $\because$ x^2+px+1 is a factor of the expression ax^3+bx+c, then for some value of k (constant)

$$ax^3+bx+c = (x^2+px+1)(ax+k)$$

$$= ax^3+(ap+k)x^2+(kp+a)x+k$$

On Comparing,

$$ap+k=0 \quad —(i)$$

$$a+pk=b \quad —(ii)$$

$$c=k \quad —(iii)$$

from (i) and (iii) $ap+c=0 \Rightarrow p=-\frac{c}{a}$

from (ii) $a-\frac{c^2}{a}=b$

$a^2-c^2=ab$ Ans (3)

13. If x is real, then the maximum and minimum values of the expression $\dfrac{x^2-3x+4}{x^2+3x+4}$ will be

(1) 2, 1 (2) 5, $\frac{1}{5}$

(3) 7, $\frac{1}{7}$ (4) None of these

Soln. Let $y = \dfrac{x^2 - 3x + 4}{x^2 + 3x + 4}$

$\Rightarrow (y-1)x^2 + 3(y+1)x + 4(y-1) = 0$

$\therefore$ for real x, $D \geq 0$

$\Rightarrow 9(y+1)^2 - [4(y-1)]^2 \geq 0$

$\Rightarrow [3(y+1) + 4(y-1)]\,[3(y+1) - 4(y-1)] \geq 0$

$\Rightarrow [7y-1]\,[-y+7] \geq 0$

$\Rightarrow \left(y - \frac{1}{7}\right)(y-7) \leq 0$

$\Rightarrow \frac{1}{7} \leq y \leq 7$ Ans $7, \frac{1}{7}$

14. If x is real, then the value of $x^2 - 6x + 13$ will not be less than

(1) 4 (2) 6 (3) 7 (4) 8

Soln. $x^2 - 6x + 13 = x^2 - 6x + 9 + 4$

$= (x-3)^2 + 4$

Clearly, the minimum value of expression is 4.

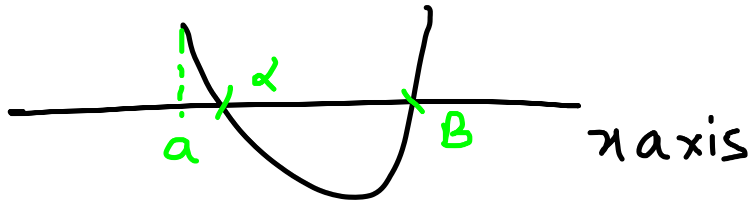
15. If the roots of $x^2 + x + a = 0$ exceeds a, then

(1) $2 < a < 3$ (2) $a > 3$

(3) $-3 < a < 3$ (4) $a < -2$

Soln. $\because$ Coefficient of x^2 in x^2+x+a is Positive. Hence graph of it must be vertically upward

$\because$ both roots are exceeds a,



$\therefore f(a) > 0$, where $f(x) = x^2+x+a$

$\Rightarrow a^2+a+a > 0 \Rightarrow a^2+2a > 0$

$\Rightarrow a(a+2) > 0$

$\Rightarrow a < -2$ or $x > 0$ —— (i)

and $D \geq 0 \Rightarrow (1)^2 - 4a \geq 0 \Rightarrow a \leq \frac{1}{4}$ —— (ii)

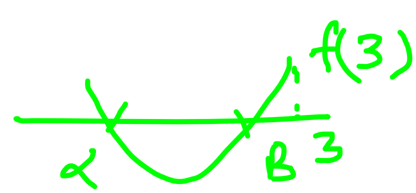
from (i) and (ii), $a < -2$

16. If the roots of the equation $x^2 - 2ax + a^2 + a - 3 = 0$ are real and less than 3, then

(1) $a < 2$ (2) $2 \leq a \leq 3$

(3) $3 < a \leq 4$ (4) $a > 4$

Soln. from Graphs $f(3) > 0$



$f(x) = x^2 - 2ax + a^2 + a - 3$

$f(3) > 0 \Rightarrow 9 - 6a + a^2 + a - 3 > 0$

$\Rightarrow a^2 - 5a + 6 > 0 \Rightarrow (a-2)(a-3) > 0$

$\Rightarrow a < 2$ or $a > 3$ —— (i)

and $D \geq 0 \Rightarrow (2a)^2 - 4 \times (a^2+a-3) \geq 0$

$\Rightarrow \cancel{4a^2} - \cancel{4a^2} - 4a + 12 \geq 0$

$\Rightarrow a \leq 3$ —— (ii)

from (i) and (ii), we get $a < 2$

17. If x be real, the least value of $x^2 - 6x + 10$ is

(1) 1 (2) 2 (3) 3 (4) 10

Soln: $x^2 - 6x + 10 = x^2 - 6x + 9 + 1$

$= (x-3)^2 + 1$

$\therefore$ least value $= 1$

18. let α, β be the roots of $x^2 + (3-\lambda)x - \lambda = 0$. The value of λ for which $\alpha^2 + \beta^2$ is minimum, is

(1) 0 (2) 1 (3) 2 (4) 3

Soln: $\because \alpha, \beta$ are the roots of equation

$\therefore$ Sum of roots $\alpha + \beta = \lambda - 3$

$\alpha\beta = -\lambda$

Now, $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$= (\lambda-3)^2 - 2(-\lambda)$

$= \lambda^2 - 6\lambda + 9 + 2\lambda$

$= \lambda^2 - 4\lambda + 4 + 5$

$= (\lambda-2)^2 + 5$

$\therefore\ \alpha^2+\beta^2$ is minimum when $(\lambda-2)^2=0$

i.e $\lambda = 2$

19. Let $f(x) = x^2+4x+1$, Then

(1) $f(x) > 0$, for all x

(2) $f(x) > 1$, when $x > 0$

(3) $f(x) \geq 1$, when $x \leq -4$

(4) $f(x) = f(-x)$ for all x.

Soln: $f(x) = x^2+4x+1$

when $f(x) \geq 1 \Rightarrow x^2+4x+1 \geq 1$

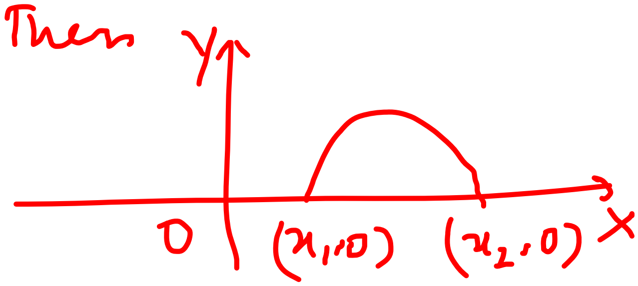
$\Rightarrow x^2+4x \geq 0$

$\Rightarrow x(x+4) \geq 0$

$\Rightarrow x \leq -4,\quad x \geq 0$

$\therefore\ f(x) \geq 1$ when $x \leq -4$ is true.

20. The figure shows the graph of $y = ax^2+bx+c$, Then



(1) $a < 0$

(2) $b^2 < 4ac$

(3) $c > 0$

(4) None of These

Soln: Since, graph is vertically downward

$\therefore$ Clearly, $a < 0$

21. If α, β be the roots of the equation $ax^2 + bx + c = 0$ and k be a real number then the condition so that $\alpha < k < \beta$ is given by

(1) $ac > 0$ (2) $ak^2 + bk + c = 0$

(3) $ac < 0$ (4) $a^2k^2 + abk + ac < 0$

Soln. Since k lies between roots of the equation. Therefore, $a f(k) < 0$ — (I)

$D \geq 0$ — (II)

Here $f(x) = ax^2 + bx + c$

when $a f(k) < 0 \Rightarrow a(ak^2 + bk + c) < 0$

$\Rightarrow a^2k^2 + abk + ac < 0$

22. If α and β $(\alpha < \beta)$ are the roots of the equation $x^2 + bx + c = 0$, where $c < 0 < b$, then

(1) $0 < \alpha < \beta$ (2) $\alpha < 0 < \beta < |\alpha|$

(3) $\alpha < \beta < 0$ (4) $\alpha < 0 < |\alpha| < \beta$

Soln. $\because$ $c < 0 < b$, it means c and

$\Rightarrow$ b are of opposite sign.

Sum of roots $= -\frac{b}{a} < 0$

$\alpha + \beta = -b < 0$ $\left(\because \begin{array}{l} a = 1 \\ b > 0 \end{array}\right)$

$\Rightarrow \quad \alpha + \beta < 0$

Product of roots $\alpha\beta = \dfrac{c}{a}$

$\alpha\beta = c < 0$

$\therefore$ α and β are of opposite sign.

But $\alpha < \beta$ $\therefore$ α is negative and β is positive

$\because \alpha + \beta < 0 \Rightarrow |\alpha| > \beta$

$\therefore$ $\alpha < 0 < \beta < |\alpha|$ is correct.

23. If α and β, α and γ, α and δ are the roots of the equations $ax^2 + 2bx + c = 0$ $2bx^2 + cx + a = 0$ and $cx^2 + ax + 2b = 0$ respectively. where a, b and c are positive real numbers, then $\alpha + \alpha^2 =$

(1) -1 (2) 0 (3) abc (4) $a+2b+c$

<u>Soln.</u> $\because$ α is a root of $ax^2 + 2bx + c = 0$ and

$a\alpha^2 + 2b\alpha + c = 0$

$\Rightarrow \quad 2b\alpha + c = -a\alpha^2$ — (1)

and $2b\alpha^2 + c\alpha + a = 0$

$(2b\alpha + c)\alpha + a = 0$ from (1)

$(-a\alpha^2)\alpha + a = 0$

$a - a\alpha^3 = 0$

$\Rightarrow \quad a(1-\alpha^3)=0 \Rightarrow a \neq 0,$

$\Rightarrow \quad 1-\alpha^3=0$

$\Rightarrow \quad (1-\alpha)(1+\alpha+\alpha^2)=0$

$\Rightarrow \quad 1-\alpha=0 \Rightarrow \alpha=1$

and $1+\alpha+\alpha^2=0 \Rightarrow \alpha=\omega, \omega^2$

But $\alpha \neq 1$ when $\alpha=1$

Then from (1) $2b+c+a=0$ which is not feasible as $a, b, c > 1$

$\therefore \alpha=\omega, \omega^2$

$\therefore \alpha+\alpha^2=\omega+\omega^2=-1$ Ans.

24. The Complete Solution of the inequation $x^2-4x<12$ is

(1) $x<-2$ or $x>6$ (2) $-6<x<2$

(3) $2<x<6$ (4) $-2<x<6$

Soln: $x^2-4x<12$

$\Rightarrow \quad x^2-4x-12<0 \Rightarrow (x-6)(x+2)<0$

$\Rightarrow -2<x<6$ Ans.

25 The set of all real value of x for which $x^2-|x+2|+x>0$ is

(1) $(-\infty,-2)\cup(2,\infty)$

(2) $(-\infty,-\sqrt{2})\cup(\sqrt{2},\infty)$

(3) $(-\infty,-1)\cup(1,\infty)$

(4) $(\sqrt{2},\infty)$

<u>soln</u> <u>Case I</u> when $x \geq -2 \Rightarrow |x+2| = x+2$

$\therefore x^2 - |x+2| + x > 0$

$\Rightarrow x^2-(x+2)+x > 0 \Rightarrow x^2-\cancel{x}-2+\cancel{x}>0$

$\Rightarrow x^2 > 2 \Rightarrow x < -\sqrt{2}$ or $x > \sqrt{2}$

But $x \geq -2 \;\therefore\; x \in (\sqrt{2},\infty)$ — (I)

Case II when $x < -2 \Rightarrow |x+2| = -(x+2)$

$\therefore x^2 - |x+2| + x > 0$

$\Rightarrow x^2 + x + 2 + x > 0$

$\Rightarrow x^2+2x+1+1 > 0 \Rightarrow (x+1)^2+1 > 0$

which is true for all real no.

But $x<-2 \;\therefore\; x < -2$ — (II)

from (I) and (II) $x \in (-\infty,-2)\cup(\sqrt{2},\infty)$

Ans (1)

26. $x^2+2ax+10-3a>0$ for all $x \in R$, then

(1) $-5<a<2$

(2) $a<-5$

(3) $a>5$

(4) $2<a<5$

Soln: * $ax^2+bx+c>0$ if $a>0$ provided $b^2-4ac<0$

$$\therefore (2a)^2-4(10-3a)<0$$

$$\Rightarrow 4a^2-4(10-3a)<0$$

$$\Rightarrow a^2-10+3a<0$$

$$\Rightarrow a^2+3a-10<0 \Rightarrow (a+5)(a-2)<0$$

$$\Rightarrow -5<a<2$$

27. If α, β, γ are the roots of the equation $x^3+x+1=0$, then the value of $\alpha^3\beta^3\gamma^3$

(1) 0 (2) -3 (3) 3 (4) -1

Soln: $\because \alpha, \beta, \gamma$ are roots of eqn.

$\therefore$ product of roots $\alpha\beta\gamma = -1$

$$\therefore \alpha^3\beta^3\gamma^3 = -1 \text{ Ans}$$

* If $ax^3+bx^2+cx+d=0$ has root α, β, γ, then.

$$\alpha+\beta+\gamma = \frac{-b}{a}, \quad \alpha\beta+\beta\gamma+\alpha\gamma = \frac{c}{a}$$

$$\alpha\beta\gamma = \frac{-d}{a}$$

28. The roots of the equation $x^4-2x^3+x = 380$ are

(1) $5, -4, \frac{1\pm 5\sqrt{-3}}{2}$ (2) $-5, 4, -\frac{1\pm 5\sqrt{-3}}{2}$

(3) $5, 4, \frac{-1 \pm 5\sqrt{-3}}{2}$ (4) $-5, -4, \frac{1 \pm 5\sqrt{-3}}{2}$

Soln: When $x = 5$, $x^4 - 2x^3 + x = 380$

$\therefore (x-5)$ is a factor of $x^4 - 2x^3 + x - 380 = 0$

$\therefore x^4 - 2x^3 + x - 380 = (x-5)(x^3 + 3x^2 + 15x + 76)$

Now $x^3 + 3x^2 + 15x + 76$ cannot be zero for any positive value of x. $\therefore$ take $x = -4$

$-64 + 48 - 60 + 76 = 0$

i.e $(x+4)$ is also factor

Two of the roots are 5 and -4

$\therefore$ Ans (1)

More explanation:- (To get other two roots)

$x^4 - 2x^3 + x - 380 = (x-5)(x+4)(x^2 - x + 19)$

$\therefore$ when $x^2 - x + 19 = 0$

$$x = \frac{1 \pm \sqrt{1-76}}{2} = \frac{1 \pm 5\sqrt{-3}}{2}$$

Hence roots are $5, -4, \frac{1 \pm 5\sqrt{-3}}{2}$

29. If α, β, γ are the roots of the equation $2x^3 - 3x^2 + 6x + 1 = 0$ then $\alpha^2 + \beta^2 + \gamma^2$ is equal to

(1) $-\frac{15}{4}$ (2) $\frac{15}{4}$ (3) $\frac{9}{4}$ (4) 4

Soln. $\alpha+\beta+\gamma = \frac{3}{2}$, $\alpha\beta+\beta\gamma+\alpha\gamma = 3$

$\alpha\beta\gamma = -\frac{1}{2}$

$$\therefore \alpha^2+\beta^2+\gamma^2 = (\alpha+\beta+\gamma)^2 - 2(\alpha\beta+\beta\gamma+\alpha\gamma)$$

$$= \left(\frac{3}{2}\right)^2 - 2\times 3 = \frac{9}{4} - 6 = -\frac{15}{4}$$

30. The solution set of the equation $pqx^2 - (p+q)^2 x + (p+q)^2 = 0$ is

(1) $\left\{\frac{p}{q}, \frac{q}{p}\right\}$ (2) $\left\{pq, \frac{p}{q}\right\}$

(3) $\left\{\frac{q}{p}, pq\right\}$ (4) $\left\{\frac{p+q}{p}, \frac{p+q}{q}\right\}$

Soln: Ans (4)

Let solution set is $\left\{\frac{p+q}{p}, \frac{p+q}{q}\right\}$

Then Sum of Roots $= \frac{p+q}{p} + \frac{p+q}{q} = \frac{(p+q)^2}{pq}$

Product of Roots $= \frac{p+q}{p} \times \frac{p+q}{q} = \frac{(p+q)^2}{pq}$

$\therefore$ Eq^n $x^2 - (\text{Sum of Roots})x + \text{Product of Roots} = 0$

$$x^2 - \frac{(p+q)^2}{pq}x + \frac{(p+q)^2}{pq} = 0$$