

EXERCISES 2.5

Infinite Limits

Find the limits in Exercises 1–12.

1. $\lim_{x \rightarrow 0^+} \frac{1}{3x}$
2. $\lim_{x \rightarrow 0^-} \frac{5}{2x}$
3. $\lim_{x \rightarrow 2^-} \frac{3}{x-2}$
4. $\lim_{x \rightarrow 3^+} \frac{1}{x-3}$
5. $\lim_{x \rightarrow -8^+} \frac{2x}{x+8}$
6. $\lim_{x \rightarrow -5^-} \frac{3x}{2x+10}$
7. $\lim_{x \rightarrow 7} \frac{4}{(x-7)^2}$
8. $\lim_{x \rightarrow 0} \frac{-1}{x^2(x+1)}$
9. a. $\lim_{x \rightarrow 0^+} \frac{2}{3x^{1/3}}$
- b. $\lim_{x \rightarrow 0^-} \frac{2}{3x^{1/3}}$
10. a. $\lim_{x \rightarrow 0^+} \frac{2}{x^{1/5}}$
- b. $\lim_{x \rightarrow 0^-} \frac{2}{x^{1/5}}$
11. $\lim_{x \rightarrow 0} \frac{4}{x^{2/5}}$
12. $\lim_{x \rightarrow 0} \frac{1}{x^{2/3}}$

Find the limits in Exercises 13–16.

13. $\lim_{x \rightarrow (\pi/2)^-} \tan x$
14. $\lim_{x \rightarrow (-\pi/2)^+} \sec x$
15. $\lim_{\theta \rightarrow 0^-} (1 + \csc \theta)$
16. $\lim_{\theta \rightarrow 0} (2 - \cot \theta)$

Additional Calculations

Find the limits in Exercises 17–22.

17. $\lim_{x \rightarrow 2} \frac{1}{x^2 - 4}$ as
 - a. $x \rightarrow 2^+$
 - b. $x \rightarrow 2^-$
 - c. $x \rightarrow -2^+$
 - d. $x \rightarrow -2^-$
18. $\lim_{x \rightarrow 1} \frac{x}{x^2 - 1}$ as
 - a. $x \rightarrow 1^+$
 - b. $x \rightarrow 1^-$
 - c. $x \rightarrow -1^+$
 - d. $x \rightarrow -1^-$
19. $\lim_{x \rightarrow 0} \left(\frac{x^2}{2} - \frac{1}{x} \right)$ as
 - a. $x \rightarrow 0^+$
 - b. $x \rightarrow 0^-$
 - c. $x \rightarrow \sqrt[3]{2}$
 - d. $x \rightarrow -1$
20. $\lim_{x \rightarrow -4} \frac{x^2 - 1}{2x + 4}$ as
 - a. $x \rightarrow -2^+$
 - b. $x \rightarrow -2^-$
 - c. $x \rightarrow 1^+$
 - d. $x \rightarrow 0^-$

21. $\lim_{x \rightarrow 0} \frac{x^2 - 3x + 2}{x^3 - 2x^2}$ as
 - a. $x \rightarrow 0^+$
 - b. $x \rightarrow 2^+$
 - c. $x \rightarrow 2^-$
 - d. $x \rightarrow 0$
 - e. What, if anything, can be said about the limit as $x \rightarrow 0$?
22. $\lim_{x \rightarrow 0} \frac{x^2 - 3x + 2}{x^3 - 4x}$ as
 - a. $x \rightarrow 2^+$
 - b. $x \rightarrow -2^+$
 - c. $x \rightarrow 0^-$
 - d. $x \rightarrow 1^+$
 - e. What, if anything, can be said about the limit as $x \rightarrow 0$?

Find the limits in Exercises 23–26.

23. $\lim_{t \rightarrow 0} \left(2 - \frac{3}{t^{1/3}} \right)$ as
 - a. $t \rightarrow 0^+$
 - b. $t \rightarrow 0^-$
24. $\lim_{t \rightarrow 0} \left(\frac{1}{t^{3/5}} + 7 \right)$ as
 - a. $t \rightarrow 0^+$
 - b. $t \rightarrow 0^-$
25. $\lim_{x \rightarrow 1} \left(\frac{1}{x^{2/3}} + \frac{2}{(x-1)^{2/3}} \right)$ as
 - a. $x \rightarrow 0^+$
 - b. $x \rightarrow 0^-$
 - c. $x \rightarrow 1^+$
 - d. $x \rightarrow 1^-$
26. $\lim_{x \rightarrow 1} \left(\frac{1}{x^{1/3}} - \frac{1}{(x-1)^{4/3}} \right)$ as
 - a. $x \rightarrow 0^+$
 - b. $x \rightarrow 0^-$
 - c. $x \rightarrow 1^+$
 - d. $x \rightarrow 1^-$

Graphing Rational Functions

Graph the rational functions in Exercises 27–38. Include the graphs and equations of the asymptotes and dominant terms.

27. $y = \frac{1}{x-1}$
28. $y = \frac{1}{x+1}$
29. $y = \frac{1}{2x+4}$
30. $y = \frac{-3}{x-3}$
31. $y = \frac{x+3}{x+2}$
32. $y = \frac{2x}{x+1}$
33. $y = \frac{x^2}{x-1}$
34. $y = \frac{x^2+1}{x-1}$
35. $y = \frac{x^2-4}{x-1}$
36. $y = \frac{x^2-1}{2x+4}$
37. $y = \frac{x^2-1}{x}$
38. $y = \frac{x^3+1}{x^2}$

Inventing Graphs from Values and Limits

In Exercises 39–42, sketch the graph of a function $y = f(x)$ that satisfies the given conditions. No formulas are required—just label the coordinate axes and sketch an appropriate graph. (The answers are not unique, so your graphs may not be exactly like those in the answer section.)

39. $f(0) = 0$, $f(1) = 2$, $f(-1) = -2$, $\lim_{x \rightarrow -\infty} f(x) = -1$, and

$$\lim_{x \rightarrow \infty} f(x) = 1$$

40. $f(0) = 0$, $\lim_{x \rightarrow \pm\infty} f(x) = 0$, $\lim_{x \rightarrow 0^+} f(x) = 2$, and

$$\lim_{x \rightarrow 0^-} f(x) = -2$$

41. $f(0) = 0$, $\lim_{x \rightarrow \pm\infty} f(x) = 0$, $\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow -1^+} f(x) = \infty$,

$$\lim_{x \rightarrow 1^+} f(x) = -\infty, \text{ and } \lim_{x \rightarrow -1^-} f(x) = -\infty$$

42. $f(2) = 1$, $f(-1) = 0$, $\lim_{x \rightarrow \infty} f(x) = 0$, $\lim_{x \rightarrow 0^+} f(x) = \infty$,

$$\lim_{x \rightarrow 0^-} f(x) = -\infty, \text{ and } \lim_{x \rightarrow -\infty} f(x) = 1$$

Inventing Functions

In Exercises 43–46, find a function that satisfies the given conditions and sketch its graph. (The answers here are not unique. Any function that satisfies the conditions is acceptable. Feel free to use formulas defined in pieces if that will help.)

43. $\lim_{x \rightarrow \pm\infty} f(x) = 0$, $\lim_{x \rightarrow 2^-} f(x) = \infty$, and $\lim_{x \rightarrow 2^+} f(x) = \infty$

44. $\lim_{x \rightarrow \pm\infty} g(x) = 0$, $\lim_{x \rightarrow 3^-} g(x) = -\infty$, and $\lim_{x \rightarrow 3^+} g(x) = \infty$

45. $\lim_{x \rightarrow -\infty} h(x) = -1$, $\lim_{x \rightarrow \infty} h(x) = 1$, $\lim_{x \rightarrow 0^-} h(x) = -1$, and

$$\lim_{x \rightarrow 0^+} h(x) = 1$$

46. $\lim_{x \rightarrow \pm\infty} k(x) = 1$, $\lim_{x \rightarrow 1^-} k(x) = \infty$, and $\lim_{x \rightarrow 1^+} k(x) = -\infty$

The Formal Definition of Infinite Limit

Use formal definitions to prove the limit statements in Exercises 47–50.

47. $\lim_{x \rightarrow 0} \frac{-1}{x^2} = -\infty$

48. $\lim_{x \rightarrow 0} \frac{1}{|x|} = \infty$

49. $\lim_{x \rightarrow 3} \frac{-2}{(x-3)^2} = -\infty$

50. $\lim_{x \rightarrow -5} \frac{1}{(x+5)^2} = \infty$

Formal Definitions of Infinite One-Sided Limits

51. Here is the definition of **infinite right-hand limit**.

We say that $f(x)$ approaches infinity as x approaches x_0 from the right, and write

$$\lim_{x \rightarrow x_0^+} f(x) = \infty,$$

if, for every positive real number B , there exists a corresponding number $\delta > 0$ such that for all x

$$x_0 < x < x_0 + \delta \quad \Rightarrow \quad f(x) > B.$$

Modify the definition to cover the following cases.

a. $\lim_{x \rightarrow x_0^-} f(x) = \infty$

b. $\lim_{x \rightarrow x_0^+} f(x) = -\infty$

c. $\lim_{x \rightarrow x_0^-} f(x) = -\infty$

Use the formal definitions from Exercise 51 to prove the limit statements in Exercises 52–56.

52. $\lim_{x \rightarrow 0^+} \frac{1}{x} = \infty$

53. $\lim_{x \rightarrow 0^-} \frac{1}{x} = -\infty$

54. $\lim_{x \rightarrow 2^-} \frac{1}{x-2} = -\infty$

55. $\lim_{x \rightarrow 2^+} \frac{1}{x-2} = \infty$

56. $\lim_{x \rightarrow 1^-} \frac{1}{1-x^2} = \infty$

Graphing Terms

Each of the functions in Exercises 57–60 is given as the sum or difference of two terms. First graph the terms (with the same set of axes). Then, using these graphs as guides, sketch in the graph of the function.

57. $y = \sec x + \frac{1}{x}$, $-\frac{\pi}{2} < x < \frac{\pi}{2}$

58. $y = \sec x - \frac{1}{x^2}$, $-\frac{\pi}{2} < x < \frac{\pi}{2}$

59. $y = \tan x + \frac{1}{x^2}$, $-\frac{\pi}{2} < x < \frac{\pi}{2}$

60. $y = \frac{1}{x} - \tan x$, $-\frac{\pi}{2} < x < \frac{\pi}{2}$

Grapher Explorations—Comparing Graphs with Formulas

Graph the curves in Exercises 61–64. Explain the relation between the curve's formula and what you see.

61. $y = \frac{x}{\sqrt{4-x^2}}$

62. $y = \frac{-1}{\sqrt{4-x^2}}$

63. $y = x^{2/3} + \frac{1}{x^{1/3}}$

64. $y = \sin\left(\frac{\pi}{x^2 + 1}\right)$