Ganit Bodh Series (Conformal-06) : Trigonometry

Mathematics:Properties Of Triangles

1. If k be the Perimeter of $\triangle ABC$, then $b\cos^2\frac{C}{2} + c\cos^2\frac{B}{2}$ is equal to

(1) k (2) $2k$ (3) $\frac{k}{2}$ (4) None of these

$\because \cos\frac{B}{2} = \sqrt{\frac{s(s-b)}{ac}}$

$\cos\frac{C}{2} = \sqrt{\frac{s(s-c)}{ab}}$

$s = \frac{a+b+c}{2}$

Soln : $\cancel{b}\frac{s(s-c)}{a\cancel{b}} + \cancel{c}\frac{s(s-b)}{a\cancel{c}}$

$= \frac{s[s-c+s-b]}{a}$

$= \frac{s[2s-(b+c)]}{a}$

$= \frac{s[a+\cancel{b}+\cancel{c}-(\cancel{b}+\cancel{c})]}{a} = s = \frac{k}{2}$ Ans.

2. If R denotes Circum radius then in $\triangle ABC$, $\frac{b^2-c^2}{2aR}$ is equal to

(1) $\cos(B-C)$ (2) $\sin(B-C)$

(3) $\cos B - \cos C$ (4) None of these

Soln We know by Sine formula

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$$

$\therefore \frac{b^2-c^2}{2aR} = \frac{(2R\sin B)^2 - (2R\sin C)^2}{2\cdot 2R\sin A\cdot R}$

$= \frac{\sin^2 B - \sin^2 C}{\sin A} = \frac{\sin(B+C)\sin(B-C)}{\sin A}$

$= \frac{\sin(\pi - A)\sin(B-C)}{\sin A} = \frac{\cancel{\sin A}\sin(B-C)}{\cancel{\sin A}}$

$= \sin(B-C)$

$\because \sin(B+C)\sin(B-C) = \sin^2 B - \sin^2 C$

3. In $\triangle ABC$, $\cot\frac{A-B}{2}\tan\frac{A+B}{2}$ is equal to

(1) $\frac{a+b}{a-b}$ (2) $\frac{a-b}{a+b}$ (3) $\frac{a(a-b)}{b(a+b)}$ (4) None of these

Sol^n. $\cot\frac{A-B}{2}\tan\frac{A+B}{2} = \frac{2\sin\frac{A+B}{2}\cos\frac{A-B}{2}}{2\cos\frac{A+B}{2}\sin\frac{A-B}{2}} = \frac{\sin A+\sin B}{\sin A-\sin B}$

$= \frac{a+b}{a-b}$

$\boxed{\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R \Rightarrow \sin A = \frac{a}{2R} \text{ etc}}$

4. In a $\triangle ABC$, $(c+a+b)(a+b-c) = ab$. The measure of $\angle C$ is

(1) $\frac{\pi}{3}$ (2) $\frac{\pi}{6}$ (3) $\frac{2\pi}{3}$ (4) None of these

Sol^n $(c+a+b)(a+b-c) = ab$

$\Rightarrow (a+b)^2 - c^2 = ab \Rightarrow a^2+b^2-c^2+2ab = ab$

$\Rightarrow a^2+b^2-c^2 = -ab$ —— (1)

$\because \cos C = \frac{a^2+b^2-c^2}{2ab} = \frac{-ab}{2ab} = -\frac{1}{2}$ $\therefore C = \frac{2\pi}{3}$

5. In a $\triangle ABC$, $A:B:C = 3:5:4$. Then $a+b+c\sqrt{2}$ is equal to

(1) $2b$ (2) $2c$ (3) $3b$ (4) $3a$

Sol^n Let angles are $3x, 5x, 4x$

$\therefore 3x+5x+4x = 180^\circ \Rightarrow x = 15^\circ$

$\therefore$ Angles are $45^\circ, 75^\circ, 60^\circ$

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \Rightarrow \frac{a}{\sin 45} = \frac{b}{\sin 75} = \frac{c}{\sin 60}$

$\Rightarrow \frac{a}{\frac{1}{\sqrt{2}}} = \frac{b}{\frac{\sqrt{3}+1}{2\sqrt{2}}} = \frac{c}{\frac{\sqrt{3}}{2}} = k\text{(say)}$

$\Rightarrow \quad a = \frac{K}{\sqrt{2}}, \quad b = \left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right)K \quad \text{and} \quad c = \frac{\sqrt{3}}{2}K$

$\therefore \quad a + b + \sqrt{2}\,c = K\left[\frac{1}{\sqrt{2}} + \frac{\sqrt{3}+1}{2\sqrt{2}} + \frac{\sqrt{3}}{\sqrt{2}}\right] = \frac{3(\sqrt{3}+1)K}{2\sqrt{2}}$

$= 3b$

6. In a $\triangle ABC$, $a^2\cos^2 A = b^2 + c^2$ Then,

(1) $A < \frac{\pi}{4}$ (2) $\frac{\pi}{4} < A < \frac{\pi}{2}$

(3) $A > \frac{\pi}{2}$ (4) $A = \frac{\pi}{2}$

Sol^n $a^2\cos^2 A = b^2 + c^2 \Rightarrow a^2(1-\sin^2 A) = b^2 + c^2$

$\Rightarrow \quad a^2 - a^2\sin^2 A = b^2 + c^2$

$\Rightarrow \quad b^2 + c^2 - a^2 = -a^2\sin^2 A \quad \text{--- (1)}$

$\cos A = \frac{b^2+c^2-a^2}{2bc} = -\frac{a^2\sin^2 A}{2bc} < 0 \quad \therefore A > \frac{\pi}{2}$

7. In a $\triangle ABC$, $\tan\frac{A}{2}$ and $\tan\frac{B}{2}$ satisfy $6x^2 - 5x + 1 = 0$

Then,

(1) $a^2 + b^2 > c^2$ (2) $a^2 - b^2 = c^2$

(3) $a^2 + b^2 = c^2$ (4) None of these

Sol^n Sum of roots $= -\frac{b}{a} \Rightarrow \tan\frac{A}{2} + \tan\frac{B}{2} = \frac{5}{6}$ --- (1)

Product of roots $= \frac{c}{a} \Rightarrow \tan\frac{A}{2}\cdot\tan\frac{B}{2} = \frac{1}{6}$ --- (11)

$\therefore \tan\left(\frac{A+B}{2}\right) = \frac{\tan\frac{A}{2} + \tan\frac{B}{2}}{1 - \tan\frac{A}{2}\tan\frac{B}{2}} = \frac{\frac{5}{6}}{1-\frac{1}{6}} = \frac{\frac{5}{6}}{\frac{5}{6}} = 1$

$\therefore \frac{A+B}{2} = \frac{\pi}{4} \Rightarrow A + B = \frac{\pi}{2} \quad \therefore \angle C = \frac{\pi}{2}$

$\therefore$ ABC is right angle triangle, right angle at C

$\therefore c^2 = a^2 + b^2$

8. In a $\triangle ABC$, $a = 8$, $b = 10$ and $c = 12$, Then $\angle C$ is equal to

(1) $\frac{A}{2}$ (2) $2A$ (3) $3A$ (4) None of these

Soln. $\because \cos A = \frac{b^2+c^2-a^2}{2bc} = \frac{100+144-64}{2\times 10\times 12} = \frac{\cancel{180}}{2\times\cancel{10}\times\cancel{12}} = \frac{3}{4}$

$\cos 2A = 2\cos^2 A - 1 = 2\times\left(\frac{3}{4}\right)^2 - 1 = \cancel{2}\times\frac{9}{\cancel{16}} - 1 = \frac{1}{8}$

$\cos C = \frac{a^2+b^2-c^2}{2ab} = \frac{64+100-144}{2\times 8\times 10} = \frac{\cancel{20}}{\cancel{2}\times 8\times\cancel{10}} = \frac{1}{8}$

$= \cos 2A \quad \therefore \quad C = 2A$ Ans.

9. In a $\triangle ABC$, The perimeter $= 2s$ and the ex-radii are r_1, r_2 and r_3. Then $r_1 r_2 + r_2 r_3 + r_3 r_1$ is equal to

(1) s^2 (2) $2s^2$ (3) $3s^2$ (4) $4s^2$

Soln $\boxed{\because r_1 = \frac{\Delta}{s-a}, \quad r_2 = \frac{\Delta}{s-b}, \quad r_3 = \frac{\Delta}{s-c}}$

Now $r_1 r_2 + r_2 r_3 + r_3 r_1$

$= \frac{\Delta}{(s-a)}\frac{\Delta}{(s-b)} + \frac{\Delta}{(s-b)}\frac{\Delta}{(s-c)} + \frac{\Delta}{(s-c)(s-a)}$

$= \Delta^2\left[\frac{s-c+s-a+s-b}{(s-a)(s-b)(s-c)}\right] = \frac{\Delta^2[3s-(a+b+c)]}{(s-a)(s-b)(s-c)}$

$= \frac{\Delta^2[3s-2s]}{(s-a)(s-b)(s-c)} = \frac{\Delta^2 \cdot s\times s}{s(s-a)(s-b)(s-c)} = \frac{s^2\cancel{\Delta^2}}{\cancel{\Delta^2}} = s^2$

10. In a $\triangle ABC$, if $\tan\frac{A}{2} = \frac{5}{6}$ and $\tan\frac{B}{2} = \frac{20}{37}$, then

(1) $2a = b+c$ (2) $a > b > c$

(3) $2c = a+b$ (4) None of these

Sol^n $\tan\left(\frac{A+B}{2}\right) = \frac{\tan\frac{A}{2} + \tan\frac{B}{2}}{1-\tan\frac{A}{2}\tan\frac{B}{2}} = \frac{\frac{5}{6}+\frac{20}{37}}{1-\frac{5}{6}\times\frac{20}{37}}$

$\therefore \tan\left(\frac{\pi}{2}-\frac{C}{2}\right) = \frac{185+120}{222-100} = \frac{305}{122}$

$\cot\frac{C}{2} = \frac{305}{122}$ $\therefore \tan\frac{C}{2} = \frac{122}{305} = 0.4$

$\tan\frac{A}{2} = \frac{5}{6} = .83$, $\tan\frac{B}{2} = \frac{20}{37} = 0.54$

$\because \tan\frac{A}{2} > \tan\frac{B}{2} > \tan\frac{C}{2} \Rightarrow A > B > C$.

$\Rightarrow a > b > c$

$\therefore$ Side opposite to greater angle are greater.

11. In a ΔABC, $a=5$, $b=4$ and $\tan\frac{C}{2} = \sqrt{\frac{7}{9}}$. The side c is

(1) 6 (2) 3 (3) 2 (4) None of these

Sol^n. $\because \tan\frac{C}{2} = \sqrt{\frac{7}{9}}$ $\therefore \cos C = \frac{1-\tan^2\frac{C}{2}}{1+\tan^2\frac{C}{2}} = \frac{1-\frac{7}{9}}{1+\frac{7}{9}} = \frac{2}{16}$

$\because \frac{a^2+b^2-c^2}{2ab} = \frac{1}{8} \Rightarrow \frac{5^2+4^2-c^2}{2\times5\times4} = \frac{1}{8}$

$\Rightarrow 41-c^2 = 5 \Rightarrow c^2 = 36 \Rightarrow c = 6$

12. In a ΔABC, $B = \frac{\pi}{8}$ and $C = \frac{5\pi}{8}$, The altitude from A to side BC is

(1) $\frac{a}{2}$ (2) $2a$ (3) $\frac{1}{2}(b+c)$ (4) None

Sol^n $A = \pi - \left(\frac{\pi}{8}+\frac{5\pi}{8}\right) = \frac{\pi}{4}$

length of Altitude from A to BC

$$= 2R\left(\cos A + \cos B \cos C\right)$$

$$= 2R\left(\cos\frac{\pi}{4} + \cos\frac{\pi}{8}\cos\frac{5\pi}{8}\right)$$

$$= 2R\left[\cos\frac{\pi}{4} + \frac{1}{2}\left(\cos\frac{3\pi}{4} + \cos\frac{\pi}{2}\right)\right]$$

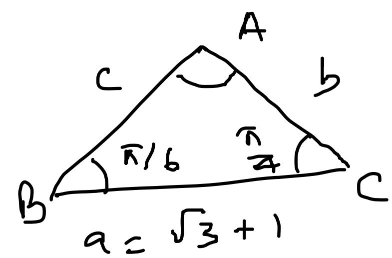
$$= 2R\left[\cos\frac{\pi}{4} + \frac{1}{2}\cos\frac{3\pi}{4}\right] = 2R\left(\frac{1}{\sqrt{2}} - \frac{1}{2}\frac{1}{\sqrt{2}}\right)$$

$$= 2R \times \frac{1}{2\sqrt{2}} = \frac{1}{2}\left(2R\sin A\right) = \frac{a}{2} \quad \underline{\text{Ans.}} \quad \left[\because A = \frac{\pi}{4}\right]$$

13. The angles of a triangle are $\frac{\pi}{6}$ and $\frac{\pi}{4}$ and the length of included side is $(\sqrt{3}+1)$ cm. The area of the triangle is

(1) $\frac{\sqrt{3}-1}{2}\ cm^2$ (2) $\frac{\sqrt{3}}{2}\ cm^2$

(3) $\frac{\sqrt{3}+1}{2}\ cm^2$ (4) None of these



Soln

$$A = \pi - \left(\frac{\pi}{6} + \frac{\pi}{4}\right) = \frac{7\pi}{12}$$

By Sine formula,

$$\frac{a}{\sin A} = \frac{b}{\sin B} \Rightarrow b = \frac{\sin B}{\sin A}\, a.$$

$$\therefore b = \frac{\frac{1}{2}}{\frac{\sqrt{3}+1}{2\sqrt{2}}}\left(\sqrt{3}+1\right) = \sqrt{2}$$

$$\therefore \text{Area of } \Delta = \frac{1}{2}ab\sin C = \frac{1}{2}\left(\sqrt{3}+1\right)\sqrt{2} \times \sin\frac{\pi}{4}$$

$$= \frac{(\sqrt{3}+1)}{2} \times \cancel{\sqrt{2}} \times \frac{1}{\cancel{\sqrt{2}}} = \frac{\sqrt{3}+1}{2} \text{ Sq. cm.}$$

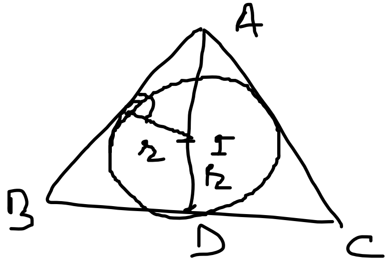
14. If in ΔABC, the incentre of the middle point of the median AD, then $\cos A$ has the value.

(1) $\frac{7}{8}$ (2) $\frac{1}{4}$ (3) $\frac{1}{3}$ (4) doesn't exist

Soln: ∵ I be the incentre ∴ ID = r

∴ If I be the mid-point of AD

∴ AD = 2r which is not possible for inradius. Hence, such Δ doesn't exist.



15. If in the Δ ABC, 3a = b+c, then $\tan\frac{B}{2}\tan\frac{C}{2}$ is equal to

(1) $\tan\frac{A}{2}$ (2) 1 (3) 2 (4) None of these

Soln

$$\tan\frac{B}{2}\tan\frac{C}{2} = \sqrt{\frac{(s-a)\cancel{(s-c)}}{s\cancel{(s-b)}} \times \frac{(s-a)\cancel{(s-b)}}{s\cancel{(s-c)}}} = \frac{s-a}{s}$$

$$= \frac{\frac{a+b+c-a}{2}}{s} = \frac{b+c-a}{2s} = \frac{b+c-a}{b+c+a}$$

$$= \frac{3a-a}{3a+a} = \frac{2a}{4a} = \frac{1}{2} \quad [\because b+c=3a]$$

16. If in the Δ ABC, 2 cos A sin C = sin B, then the triangle is

(1) equilateral (2) isosceles

(3) right angled (4) None of these

Soln

$$2\cos A\sin C = \sin B$$

$$\sin(A+C) - \sin(A-C) = \sin B$$

$$\sin(\pi-B) - \sin(A-C) = \sin B$$

$$\cancel{\sin B} - \sin(A-C) = \cancel{\sin B} \Rightarrow \sin(A-C) = 0$$

$$\Rightarrow A - C = 0$$

$$\Rightarrow A = C$$

∴ Δ is isosceles.

17. In a Δ ABC, a = 1, the perimeter is six times the AM of sines of angles. The measures of ∠A is

(1) $\frac{\pi}{3}$ (2) $\frac{\pi}{2}$ (3) $\frac{\pi}{6}$ (4) $\frac{\pi}{4}$

Sol^n $\because\ a+b+c = \overset{2}{\cancel{6}}\left(\dfrac{\sin A+\sin B+\sin C}{\cancel{3}}\right)$

$\Rightarrow \dfrac{1+b+c}{2} = \sin A + \sin B + \sin C$

$\Rightarrow \dfrac{1+b+c}{2} = \sin A + b\sin A + c\sin A$

$\Rightarrow \dfrac{(1+b+c)}{2} = (1+b+c)\sin A$

$\Rightarrow \sin A = \dfrac{1}{2} \Rightarrow A = \dfrac{\pi}{6}$ Ans.

$\boxed{\begin{array}{l} \dfrac{a}{\sin A} = \dfrac{b}{\sin B} \\ \therefore \sin B = b\sin A \\ \sin C = c\sin A \\ \because a = 1 \end{array}}$

18. In $\triangle ABC$, $c\cos^2\dfrac{A}{2} + a\cos^2\dfrac{C}{2} = \dfrac{3b}{2}$, then a, b, c are in

(1) G.P (2) H.P. (3) A.P (4) None

Sol^n $c\cos^2\dfrac{A}{2} + a\cos^2\dfrac{C}{2} = \dfrac{3b}{2}$

$\boxed{\because 1+\cos A = 2\cos^2\dfrac{A}{2}}$

$c\left(\dfrac{1+\cos A}{2}\right) + a\left(\dfrac{1+\cos C}{2}\right) = \dfrac{3b}{2}$

$\Rightarrow \dfrac{1}{2}\left[a + c + c\cos A + a\cos C\right] = \dfrac{3b}{2}$

$\Rightarrow \dfrac{1}{2}\left[a + c + b\right] = \dfrac{3b}{2}$

$\boxed{\because b = a\cos C + c\cos A}$

$\Rightarrow a+b+c = 3b \Rightarrow a+c = 2b$

$\therefore a, b, c$ are in A P

19. The ratio of the distance of the orthocentre of an acute angled triangle ABC from the sides BC, CA and AB is

(1) $\cos A : \cos B : \cos C$

(2) $\sin A : \sin B : \sin C$

(3) $\sec A : \sec B : \sec C$

(4) None of these

Sol^n. $\because$ Distance of orthocentre from vertices A, B, C are respectively $2R\cos A$, $2R\cos B$, $2R\cos C$.

$\therefore$ Required ratio $\cos A : \cos B : \cos C$

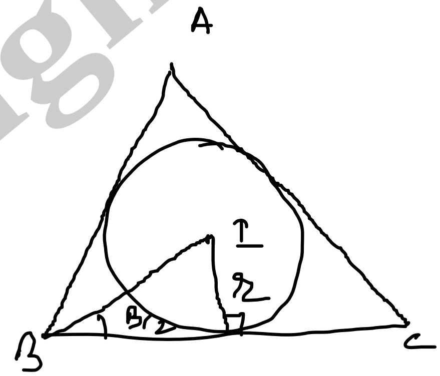
20. In a $\triangle ABC$, I is incentre. The ratio $IA : IB : IC$ is equal to

(1) $\operatorname{cosec}\frac{A}{2} : \operatorname{cosec}\frac{B}{2} : \operatorname{cosec}\frac{C}{2}$

(2) $\sin\frac{A}{2} : \sin\frac{B}{2} : \sin\frac{C}{2}$

(3) $\sec\frac{A}{2} : \sec\frac{B}{2} : \sec\frac{C}{2}$

(4) None of these



Sol^n If I be incentre.

$\therefore BI = r\operatorname{cosec}\frac{B}{2}$

Similarly, $AI = r\operatorname{cosec}\frac{A}{2}$, $CI = r\operatorname{cosec}\frac{C}{2}$

$\therefore AI : BI : CI = \operatorname{cosec}\frac{A}{2} : \operatorname{cosec}\frac{B}{2} : \operatorname{cosec}\frac{C}{2}$

21. In a $\triangle ABC$, the sides are in ratio $4 : 5 : 6$. The ratio of circumradius and the inradius is

(1) $8 : 7$ (2) $3 : 2$ (C) $7 : 3$ (D) $16 : 7$

Sol^n. $\because R = \frac{abc}{4\Delta}$, $r = \frac{\Delta}{s}$, $s = \frac{a+b+c}{2} = \frac{15}{2}$

$\therefore \Delta = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{\frac{15}{2}\times\frac{7}{2}\times\frac{5}{2}\times\frac{3}{2}} = \frac{15}{4}\sqrt{7}$

$\therefore R : r = \frac{abc}{4\Delta} : \frac{\Delta}{s} = \frac{4\times5\times6}{\cancel{4}\times\frac{15}{\cancel{4}}\sqrt{7}} : \frac{\frac{15}{4}\sqrt{7}}{\frac{15}{2}}$

$= \frac{8}{\sqrt{7}} : \frac{\sqrt{7}}{2} = 16 : 7$ Ans.

22. In a ΔABC, R = Circumradius, r = inradius.

The value of $\dfrac{a\cos A + b\cos B + c\cos C}{a+b+c}$ is equal to

(1) $\dfrac{R}{r}$ (2) $\dfrac{R}{2r}$ (3) $\dfrac{r}{R}$ (4) $\dfrac{2r}{R}$

Soln:

$$\boxed{\text{We know } \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R}$$

$\therefore a = 2R\sin A$ etc

$$\therefore \frac{a\cos A + b\cos B + c\cos C}{a+b+c}$$

$$= \frac{2R\sin A\cos A + 2R\sin B\cos B + 2R\sin C\cos C}{a+b+c}$$

$$= \frac{\cancel{R}(\sin 2A + \sin 2B + \sin 2C)}{2\cancel{R}(\sin A + \sin B + \sin C)} = \frac{\cancel{4}\sin A\sin B\sin C}{2\times\cancel{4}\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}}$$

$$= \frac{2\sin A/2\cos A/2\ 2\sin B/2\cos B/2\ 2\sin C/2\cos C/2}{2\cos A/2\ \cos B/2\ \cos C/2}$$

$$= 4\sin A/2\ \sin B/2\ \sin C/2 = \frac{r}{R}$$

$\because r = 4R\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$

$\sin 2A + \sin 2B + \sin 2C = 4\sin A\sin B\sin C$

$\sin A + \sin B + \sin C = 4\cos A/2\cos B/2\cos C/2$

using $\sin C + \sin C$ it can be easily derive

23. In a ΔABC, $2s$ = Perimeter and R = Circumradius

Then $\frac{s}{R}$ is equal to

(1) $\sin A + \sin B + \sin C$ (2) $\cos A + \cos B + \cos C$

(3) $\sin\frac{A}{2} + \sin\frac{B}{2} + \sin\frac{C}{2}$ (4) None of these

Sol^n. $\frac{s}{R} = \frac{a+b+c}{2R} = \frac{a}{2R} + \frac{b}{2R} + \frac{c}{2R}$

$= \sin A + \sin B + \sin C$ (by Sine formula)

24. The ratio of circumradius and inradius of an equilateral $\triangle$ is

(1) 3 : 1 (2) 1 : 1 (3) 2 : $\sqrt{3}$ (4) 2 : 1

Sol^n $\because r = 4R \sin\frac{A}{2} \sin\frac{B}{2} \sin\frac{C}{2}$

$\frac{r}{R} = \cancel{4} \times \frac{1}{\cancel{2}} \times \frac{1}{\cancel{2}} \times \frac{1}{2}$ $(\because A = B = C = 60^\circ)$

$\therefore R = 2r = \frac{R}{r} = 2 : 1$

25. If in a $\triangle ABC$, $a^2 + b^2 + c^2 = 8R^2$, where R is circum radius, then the triangle is

(1) equilateral (2) isosceles

(3) right angled (4) None of these

Sol^n $\because$ we know $\boxed{\frac{a}{\sin A} = \frac{b}{\sin A} = \frac{c}{\sin C} = 2R}$

$\therefore a = 2R\sin A$ etc.

$\therefore a^2 + b^2 + c^2 = 8R^2$

$(2R\sin A)^2 + (2R\sin B)^2 + (2R\sin C)^2 = 8R^2$

$4R^2(\sin^2 A + \sin^2 B + \sin^2 C) = 8R^2$

$\sin^2 A + \sin^2 B + \sin^2 C = 2$

$\sin^2 A + 1 - \cos^2 B + \sin^2 C = 2$

$\sin^2 A - (\cos^2 B - \sin^2 C) = 2 - 1$

$\sin^2 A - \cos(B+C)\cos(B-C) = 1$

$\Rightarrow -\cos(\pi - A)\cos(B-C) = 1 - \sin^2 A$

$\Rightarrow \cos A \cos(B-C) - \cos^2 A = 0$

$\Rightarrow \cos A \; [\cos(B-C) - \cos A] = 0$

$\Rightarrow \cos A \; [\cos(B-C) + \cos(B+C)] = 0$

$\Rightarrow 2 \cos A \cos B \cos C = 0$

i.e. $\cos A = 0$ or $\cos B = 0$ or $\cos C = 0$

$\therefore$ One of the angle must be right angle.

26. In an equilateral $\triangle$, circumradius : inradius : exradius is equal to

(1) 1 : 1 : 1 (2) 1 : 2 : 3

(3) 2 : 1 : 3 (4) 3 : 2 : 4

Sol^n. $r = 4R \sin\frac{A}{2} \sin\frac{B}{2} \sin\frac{C}{2}$ $(\because A = B = C = 60^\circ)$

$= \cancel{4}R \times \frac{1}{\cancel{2}} \times \frac{1}{\cancel{2}} \times \frac{1}{2}$

$\therefore \frac{R}{2} = r$ —— (1)

Also, $r_1 = 4R \sin\frac{A}{2} \cos\frac{B}{2} \cos\frac{C}{2}$

$= \cancel{4}R \times \frac{1}{2} \times \frac{\sqrt{3}}{\cancel{2}} \times \frac{\sqrt{3}}{\cancel{2}} \Rightarrow r_1 = \frac{3R}{2}$

$\therefore \frac{R}{2} = \frac{r_1}{3}$.

$\therefore$ Circumradius : in radius : ex radius.

$= R : r : r_1$

$= R : \frac{R}{2} : \frac{3R}{2} = 2 : 1 : 3$ Ans.

27. If in a triangle. R and r are circumradius and inradius respectively. Then the HM of the exradii of the triangle is

(1) $3r$ (2) $2R$ (c) $R+r$ (4) None.

Sol^n. We know, $r = \frac{\Delta}{s}$, $r_1 = \frac{\Delta}{s-a}$, $r_2 = \frac{\Delta}{s-b}$, $r_3 = \frac{\Delta}{s-c}$

HM of exradii $= \dfrac{3}{\frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}} = \dfrac{3}{\frac{s-a}{\Delta}+\frac{s-b}{\Delta}+\frac{s-c}{\Delta}}$

$= \dfrac{3\Delta}{3s-(a+b+c)} = 3\cdot\dfrac{\Delta}{3s-2s} = 3\dfrac{\Delta}{s}$

$= 3r \quad \because \boxed{a+b+c=2s}$

28. In a ΔABC, the inradius and three exradii are r, r_1, r_2, r_3 respectively. In usual notations the value of $r\cdot r_1\cdot r_2\cdot r_3$ is equal to

(1) 2Δ (2) Δ^2 (3) $\dfrac{abc}{4R}$ (4) None.

Soln. we know, $r = \dfrac{\Delta}{s}$, $r_1 = \dfrac{\Delta}{s-a}$, $r_2 = \dfrac{\Delta}{s-b}$, $r_3 = \dfrac{\Delta}{s-c}$

$\therefore r\cdot r_1\cdot r_2\cdot r_3 = \dfrac{\Delta^4}{s(s-a)(s-b)(s-c)} = \dfrac{\Delta^4}{\Delta^2} = \Delta^2$

$\boxed{\because \Delta = \sqrt{s(s-a)(s-b)(s-c)}}$

29. Let A_0, A_1, A_2, A_3, A_4 and A_5 be the consecutive vertices of a regular hexagon inscribed in a unit circle. The product of the length A_0A_1, A_0A_2 and A_0A_4 is

(A) $\dfrac{3}{4}$ (B) $3\sqrt{3}$ (C) 3 (D) $\dfrac{3\sqrt{3}}{2}$

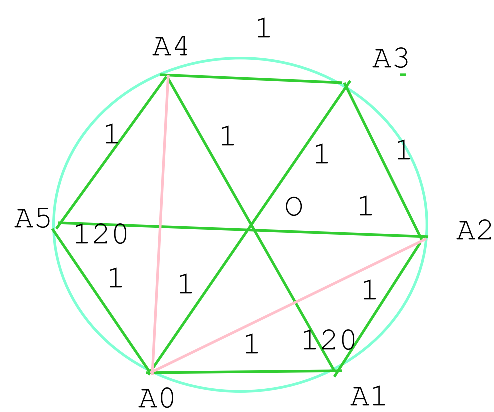
Soln. If O be the centre

$\therefore \angle A_0OA_1 = \dfrac{360}{6} = 60^\circ$

$\therefore OA_0 = OA_1$

$\therefore$ Opposite angles are equal

$\therefore A_0OA_1$ is equilateral Δ.



$A_0A_1 = 1$ —— (1)

$\angle A_0 A_1 A_2 = 120^\circ$

using cosine formula

$A_0A_2 = \sqrt{1^2 + 1^2 - 2\times 1\times 1 \cos 120^\circ} = \sqrt{1+1+\cancel{2}\times\frac{1}{\cancel{2}}}$

$= \sqrt{3}$

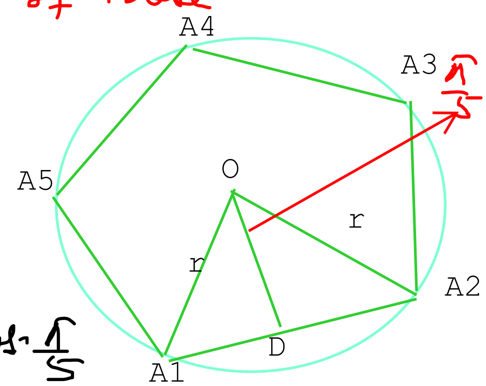
$A_0A_4 = A_0A_2 = \sqrt{3}$ —— (2)

$\therefore A_0A_1 \times A_0A_2 \times A_0A_4 = 1\times\sqrt{3}\times\sqrt{3} = 3$

30. The area of circle is A_1 and the area of regular pentagon inscribed in the circle is A_2. Then $A_1 : A_2$ is

(1) $\frac{\pi}{5}\cos\frac{\pi}{10}$ (2) $\frac{2\pi}{5}\sec\frac{\pi}{10}$

(3) $\frac{2\pi}{5}\operatorname{cosec}\frac{\pi}{10}$ (4) None of these



Sol^n Let O be the centre of circle.

$\therefore \angle A_1 O A_2 = \frac{2\pi}{5}$

$OD \perp A_1A_2 \therefore A_1OD = \frac{\pi}{5}$

$\therefore A_1A_2 = 2r\sin\frac{\pi}{5}$. $OD = r\cos\frac{\pi}{5}$

$\therefore$ Area of Pentagon $A_2 = 5\times$ Area of $\triangle A_1OA_2$

$A_2 = 5\times\frac{1}{2}\times 2r\sin\frac{\pi}{5}\, r\cos\frac{\pi}{5}$

$A_2 = \frac{5}{2}r^2\sin\frac{2\pi}{5}$

$\therefore$ Area of circle $A_1 = \pi r^2$

$\therefore$ Required ratio $A_1 : A_2 = \pi\cancel{r^2} : \frac{5}{2}\cancel{r^2}\sin\frac{2\pi}{5}$

$= \frac{2\pi}{5} : \sin\frac{2\pi}{5}$ Ans (4)

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