EXERCISE

OBJECTIVE

1. Let $f(x) = |x-1|$ Then

(a) $f(x^2) = \left(f(x)^2\right)$ (b) $f(x+y) = f(x) + f(y)$

(C) $f(|x|) = |f(x)|$ (d) None of these.

2. If $f(x) = \cos(\ln x)$, then $f(x)f(y) - \frac{1}{2}[f(x/y) + f(xy)]$ equals

(a) –1 (b) 1/2

(c) –2 (d) None of these.

3. The domain of definition of the function $y = \frac{1}{\log_{10}(1-x)} + \sqrt{x+2}$ is

(a) (–3, –2) excluding (–2.5) (b) [0, 1] excluding 0.5

(c) (–2, 1) excluding 0 (d) None of these.

4. If $g(f(x)) = |\sin x|$ and $f\left(g(x)\right) = \left(\sin\sqrt{x}\right)^2$ then

(a) $f(x) = \sin^2 x \quad g(x) = \sqrt{x}$ (b) $f(x) = \sin x \quad g(x) = |x|$

(c) $f(x) = x^2 \quad g(x) = \sin\sqrt{x}$ (d) cannot be determined.

5. If $f : [1, \infty) \longrightarrow [2, \infty)$ is given by $f(x) = x + \frac{1}{x}$, then $f^{-1}(x)$ equals

(a) $\frac{x + \sqrt{x^2-4}}{2}$ (b) $\frac{x - \sqrt{x^2-4}}{2}$

(c) $\frac{x}{1+x^2}$ (d) $1 + \sqrt{x^2-4}$.

6. The domain of definition of $f(x) = \frac{\log_2(x+3)}{x^2+3x+2}$ is

(a) $IR \setminus \{-1, -2\}$ (b) $IR \setminus \{-1, -2, -3\}$

(c) (–2, ∞) (d) (–3, ∞) \ {–1, –2}

7. Domain of definition of the function $f(x)=\sqrt{\sin^{-1}(2x)+\frac{\pi}{6}}$ for real x, is

(a) $\left[\frac{-1}{4},\frac{1}{2}\right]$ (b) $\left(\frac{-1}{2},\frac{1}{9}\right)$

(c) $\left[-\frac{1}{2},\frac{1}{2}\right]$ (d) $\left[-\frac{1}{4},\frac{1}{4}\right]$.

8. Let $g(x)=1+x-[x]$ and $f(x)=\begin{Bmatrix}-1 & x<0\\ 0 & x=0\\ 1 & x>0\end{Bmatrix}$ then for all x, $f\left(g(x)\right)$ equals

(a) x (b) $f(x)$

(c) 1 (d) $g(x)$.

9. If the function $f:[1,\infty)\to[1,\infty)$ is defined by $f(x)=2^{x(x-1)}$ then $f^{-1}(x)$ is

(a) $\left(\frac{1}{2}\right)^{x(x-1)}$ (b) $\frac{1}{2}\left(1+\sqrt{1+4\log_2 x}\right)$

(c) $\frac{1}{2}\left(1+\sqrt{1-4\log_2 x}\right)$ (d) not defined.

10. Let $f(x)=(1+b^2)x^2+2bx+1$ and let $m(b)$ be the minimum value of $f(x)$. As b varies range of $m(b)$ is

(a) [0, 1] (b) [1/2, 1]

(c) $\left[0,\frac{1}{2}\right]$ (d) (0, 1].

11. If $y=f(x)=\frac{x+2}{x-1}$, then

(a) $x=f(y)$ (b) $f(1)=3$

(c) y increases with x for $x<1$ (d) f is a rational function of x.

12. If S is the set of all real x such that $\frac{2x-1}{2x^3+3x^2+x}$ is positive , then S contains

(a) $\left(-\infty,-\frac{3}{2}\right)$ (b) $\left(-\frac{1}{4},\frac{1}{2}\right)$

(c) $\left(-\frac{3}{2},\frac{-1}{4}\right)$ (d) $\left(\frac{1}{2},3\right)$.

13. Let $g(x)$ be a function defined on [–1, 1]. If the area of the equilateral triangle with two of its vertices at (0, 0) and $[x, g(x)]$ is $\sqrt{3}/4$, then the function $g(x)$ is

(a) $\sqrt{1-x^2}$ (b) $\sqrt{1+x^2}$

(c) $-\sqrt{1-x^2}$ (d) $-\sqrt{1+x^2}$.

14. If $f(x) = \cos[\pi^2]x + \cos[-\pi^2]x$, where $[x]$ stands for the GIF, then

(a) $f\left(\frac{\pi}{2}\right) = -1$ (b) $f(-\pi) = 0$

(c) $f(\pi) = 1$ (d) $f(\pi/4) = 1$.

15. For a positive integer n let $f_n(\theta) = (\tan\theta/2)(1+\sec\theta)(1+\sec 2\theta).....(1+\sec 2^n\theta)$ then

(a) $f_2(\pi/16) = -1$ (b) $f_4\left(\frac{\pi}{64}\right) = 1$

(c) $f_3\left(\frac{\pi}{32}\right) = 1$ (d) $f_5\left(\frac{\pi}{128}\right) = 1$.

SUBJECTIVE

16. Find the domain of definitions of the following functions :

(1) $f(x)=\sqrt{x-1}+\sqrt{6-x}$

(2) $f(x)=\log\frac{x^2-5x+6}{x^2+4x+6}$

(3) $f(x)=\sqrt{x^2-x-2}+\frac{1}{\sqrt{3+2x-x^2}}$

(4) $f(x)=\sin^{-1}\frac{x-3}{2}-\log(4-x)$

(5) $f(x)=\frac{x}{\sqrt{x^2-x-2}}$

(6.) $f(x)=\frac{1}{\log(1-x)}+\sqrt{x+2}$

(7) $f(x)=\sqrt{\sin x-1}$

(8) $f(x)=\log(\cos x)$

(9) $f(x)=\sqrt{\log\frac{5x-x^2}{4}}$

(10) $f(x)=\cos^{-1}\left(\frac{3}{4+2\sin x}\right)$

(11) $f(x)=\log_x 5$

(12) $f(x)=\frac{1}{\sqrt{|x|-x}}$.

(13) $f(x)=\sqrt{\sin^{-1}(\log_2 x)}$

(14) $f(x)=\frac{1}{x}+2\sin^{-1}x+\frac{1}{\sqrt{x-2}}$

(15) $f(x)=\log|4-x^2|$

(16) $f(x)=\sqrt{\cos(\sin x)}+\sin^{-1}\frac{1+x^2}{2x}$

(17) $f(x)=\frac{2x-3}{\sqrt{x^2+2x+3}}$

(18) $f(x)=\log\left(\sin(x-3)+\sqrt{16-x^2}\right.$

(19) $f(x)=\sqrt{3-x}+\cos^{-1}\left(\frac{x-2}{3}\right)$

(20) $f(x)=\frac{x}{\log(1+x)}$

(21) $f(x)=\sqrt{\sin\sqrt{x}}$

(22) $f(x)=\cos^{-1}\left(\frac{1}{\sin x}\right)$

(23) $f(x)=\sqrt{-\sin^2\pi x}$

(24) $f(x)=\sin^{-1}(|x|-3)$.

(25) $f(x)=\sqrt{\frac{x^4-3x^2+x+7}{x^4-2x^2+1}-1}$

(26) $f(x)=\frac{1}{\sin^4 x+\cos^4 x}$

(27) $f(x)=\dfrac{\log_3(x^2+1)}{\sin^2 x-\sin x+0.25}$

(28) $f(x)=\log(5x^2-8x-4)+\sqrt{x-1}$

(29) $\sqrt{\log_{0.5}(-x^2+x+6)}+\dfrac{1}{\sqrt{x^2+2x}}$

(30) $f(x)=\sqrt{16x-x^5}+\log_{½}(x^2-1)$

(31) $f(x)=\sqrt{\log_{0.5}(3x-8)-\log_{0.5}(x^2+4)}$

(32) $f(x)=\sqrt{\dfrac{3^x-4^x}{2x^2-x-8}}$

(33) $f(x)=\dfrac{\sqrt{6x-x^2-5}}{5^{x-2}-1}$

(34) $f(x)=\sqrt{\sin^2 x-\sin x}$.

(35) $f(x)=\sqrt{\log(\log x)-\log(4-\log x)-\log 3}$

(36) $f(x)=\dfrac{\log x}{\sqrt{x^2-2x-63}}$

17. Find the domain & range of the following functions

(a) $y=\dfrac{x}{|x|}$

(b) $f(x)=\log_2\left(\dfrac{\sin x-\cos x+3\sqrt{2}}{\sqrt{2}}\right)$

(c) $y=\sqrt{x-x^2}$

(d) $f(x)=\sqrt{2-x}+\sqrt{1+x}$

(e) $y=\sqrt{3x^2-4x+5}$

(f) $f(x)=\dfrac{1}{3-\cos 2x}$

(g) $y=\log(5x^2-8x+4)$

(h) $f(x)=\dfrac{x^2+2x+c}{x^2+4x+3c}$ *for* $0<c\le 1$

18. Find whether these functions are even or odd ?

(a) $f(x)=4-2x^4+\sin^2 x$

(b) $f(x)=\dfrac{1+a^{kx}}{1-a^{kx}}$

19. Find the greatest value of $f(x)=\dfrac{2}{\sqrt{2x^2-4x+3}}$

20. For what real values of *a* does the range of the function $y=\dfrac{x+1}{a+x^2}$ contain the interval [0, 1].

21. For what real values of '*a*' does the range of the function $y=\dfrac{x-1}{1-x^2-a}$ not contain any value from the interval $[-1,1]$?

22. Sketch the following graphs :

(1) $f(x) = x^4 - 2x^2 + 3$

(2) $f(x) = 2|x-2| - |x+1| + x$

(3) $f(x) = \dfrac{2x}{1+x^2}$

(4) $f(x) = \dfrac{x+3}{x+1}$

(5) $f(x) = \sin^2 x - 2\sin x$

(6) $f(x) = \sqrt{\sin x}$

(7) $f(x) = x^{1/\log x}$

(8) $f(x) = x^2 \text{signum}(x)$

(9) $f(x) = \cos + |\cos x|$

(10) $f(x) = |x+2|\, x$

(11) $f(x) = \cos^{-1}(\cos x)$

(12) $f(x) = \dfrac{1}{x^2 - 9}$

23. Prove that $\sin x^2$ is non-periodic.

24. Find the intervals in which $f(x) = 2^{\sin x} + 2^{\cos x}$ increases/decreases and find its range

25. If $r^2 - 4x + 3 = 0$ where $r = \sqrt{x^2 + y^2}$ $x, y \in$ IR, then find the greatest and least value of r.

26. What is the number of solutions to the equation $\sin[x] = \tan[x]$?

27. Find the domain and range of $f(x) = \dfrac{x^2+1}{\ln(x^2+1)}$

28. A function $f(x)$ is defined in [0, 1] satisfying $f(x) + f(y) = f\left(xy - \sqrt{1-x^2}\sqrt{1-y^2}\right)$ $\forall$ x, y and $f(0) = \pi/2$ $\quad f\left(\dfrac{1}{\sqrt{2}}\right) = \pi/4$. Find the function $f(x)$.

29. What shape will the function $f(x, y) = x^2 - y^2$ take ? (It will be a 2-dimensional surface. How will rain drops falling on the surface move along the surface ?

30. Rain drops falling from clouds are retarded with a force proportionate to their velocity. Write the distance fallen as a function of velocity i.e. $x = f(v)$.