Mental Ability

Concept-01 (Ganit Mantra Series): (Probability)

The literal meaning of probability of an event is the chance of happening of that event. So, before discussing the probability of an event, first let us discuss some points which deal with this chapter.

1. **Event:** Anything happening in this world is considered to be an event. For example, when a coin is tossed, it is considered to be an event.
2. **Outcomes:** If an event has occurred then the all possible happenings are said to be outcomes. For example, when a coin is tossed, the outcomes of this event are two, either head or tail. Similarly, when a dice is thrown, there are six possible outcomes i.e. on the face of a dice, any one of the six digits from 1 to 6 may appear and each apperanance is considered to be a different outcome than the other.
3. **Sample space:** The set of all possible outcomes of an event is called sample space. This set is denoted by S and the total number of outcomes are denoted by n(S).
4. **Favourable events:** When an event is to occur, then some of the outcomes are in favour and some of them are against our views. All those outcomes which are favouring us are called favourable events and the total number of favourable events is denoted by *n(F)*. For example, when a dice is thrown and we wish that a number must appear greater than 4. it means we wish the appearance of 5 and 6. So, out of six possible outcomes we wish only two of them. Hence number of favourable events is *n(F)* = 2 and number of all possible outcomes i.e. sample space *n(S)* = 6.

Probability of an event: Probability of an event is defined as the ratio of number of favourable event to the total number of an event, it is denoted by *P(E)*.

$$\therefore P(E) = \frac{number\ of\ \ favourable\,event}{total\ number\ of\ \ event} = \frac{n(F)}{n(S)}$$

In the above example, when a dice thrown. Then the probability that the number appear on its face is greater than 4.

Here required probability $P(E) = \frac{n(F)}{n(S)} = \frac{2}{6} = \frac{1}{3}$

Mutually exclusive event: If the happening of any one of the events in a trial prevents the happening of all others, then those events are said to be mutually exclusive. For example, the events of getting a head or a tail when a coin is tossed are mutually exclusive. If A and B are mutually exclusive events, $P(A \cap B)$ =0. Similarly, when a dice is thrown, then all its six outcomes are mutually exclusive events.

Sure events: An event which must happen is called a sure event. Probability of such an event is always 1 i.e. 100%, for example a bag contains 6 red balls and a ball is drawn from the bag. What is the probability that it is a red ball. This is a sure event.

Explanations: Since the bag contain only red balls, so when any number of balls are drawn from the bag it will always be red in colour. There is no doubt, as there are no other alternatives. Hence this event is called a sure event.

$$P(E) = \frac{number\ of\ \ red\ balls}{total\ number\ of\ \ balls} = \frac{6}{6} = 1$$

Impossible events or null events: An event which can not happen is called an impossible event. For example a bag contain 6 red balls and a ball is drawn from the bag. What is the probability that the ball drawn is black in colour. Since, the number of black (favourable) balls are zero, the probability of getting black balls $= \frac{0}{6} = 0$

Therefore, any event with zero probability are considered to be impossible events.

Exhaustive events: Two or more events are said to be exhaustive events if one of them must have happened. For example, when a coin is tossed, the appearance of head and tail are two different events, and they are considered to be exhaustive events, because either of the two events must occur when a coin is tossed.

If two events A and B are mutually exhaustive events, then $P(A \cup B)$= 1 similarly if A, B, C are exclusive events $P(A \cup B \cup C)$ = 1

If two events A and B are mutually exclusive and exhaustive events, then

$$P(A \cup B) = P(A) + P(B)$$

$$\therefore P(A) + P(B) = 1$$

$$\begin{bmatrix} \because P(A \cup B) = 1 \\ P(A \cap B) = 0 \end{bmatrix}$$

If A, B, C are any three mutually exclusive and exhaustive events, then

$$P(A \cup B \cup C) = P(A) + P(B) + P(C)$$

$$\text{and } P(A) + P(B) + P(C) = 1$$

Conditional probability: Two events are such that one of them is dependent on the other and the probability of happening of one of them depends upon the other. Then the probability of events are said to be conditional probability. For example, a card is drawn from a well shuffled pack of 52 cards and it is found to be red. What is the probability that it is a heart?

Consider the event, here two events are described. One of them that the card drawn is red in colour. Let this event be denoted by A. Now, the second event is described that it is heart. Since the card drawn is red in colour. Hence there are two possibilities that the drawn card is either a heart or a diamond. Since, the first event is confirmed, the probability of second event is defined in the sample space of first event only i.e. out of 26 red cards. We have to find the probability of getting a heart. Since hearts are 13 cards in number, hence the number of favourable cards is 13 and since we have to determine the probability out of the sample space described by the first event. Hence, total number of outcomes i.e. n(s) = 26

Hence required probability $= \dfrac{n(f)}{n(s)} \dfrac{13}{26} = \dfrac{1}{2}$

Logical support: First of all, we think of event A and its probability. So, out of 52 cards from a well shuffled pack, probability that a card drawn is red is

$$P(A) = \frac{\text{Number of red cards (favourable cards)}}{\text{Total number of cards in pack (sample space)}}$$

$$= \frac{26}{52} = \frac{1}{2}$$

Let B denote an event in which the drawn card is a heart. Now, the probability that the card is red and heart i.e.

$$P(A \cap B) = \frac{13}{52} = \frac{1}{4}$$

Now, we have to find probability of B, when A has already happened

$$P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$$

$$= \frac{\frac{1}{4}}{\frac{1}{2}} = \frac{2}{4} = \frac{1}{2}$$

P(A/B) means probability of A when B has already happened. Similarly, *P(B/A)* means probability of B when A has already happened and is defined as

$$P(B/A) = \frac{P(A \cap B)}{P(A)}$$

Let us consider another example. If a dice is thrown and it is found that the number which appeared on its faces is an even number. Then what is the probability that the number appeared is a prime number. Here also, there are two events one of them described as even number is denoted by A and other which is prime number is denoted by B.

Therefore, probability of A = $\dfrac{3}{6} = \dfrac{1}{2}$

Since only three even numbers (2, 4, 6) are there, out of six numbers marked on the faces of a dice. The probability of a prime number i.e. Probability of B = $\dfrac{3}{6} = \dfrac{1}{2}$

Since there are only three prime numbers (2, 3, 5) out of the six numbers marked on the faces of a dice.

$P(A \cap B)$ means probability of that number which is even and prime i.e. only number 2 out of the six numbers marked on the faces of a dice qualify these conditions.

Hence, $P(A \cap B) = \dfrac{1}{6}$

So by definition, probability

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{6}}{\frac{1}{2}} = \frac{1}{3}$$

That is probability of getting an even number when a prime number has already appeared on its face is

$$P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)} = \frac{\frac{1}{6}}{\frac{1}{2}} = \frac{1}{3}$$

This means that the probability of getting a prime number when it is known that an even number has already appeared on the face of a dice.

Addition law: If A and B are two different outcomes of an event

Then *n(A)* denotes all the outcomes of A and *n(B)* denotes all outcomes of B.

$n(A \cup B)$ denotes all outcomes of A or B or both

$n(A \cap B)$ denotes all those outcomes common to A and B.

n(S) denotes the number of all outcomes i.e. sample space

By set theory:

$$\therefore n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$\therefore \frac{n(A \cup B)}{n(S)} = \frac{n(A)}{n(S)} + \frac{n(S)}{n(S)} - \frac{n(A \cap B)}{n(S)}$$

$$\Rightarrow P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

i.e. Probability of happening of A or B = probability of A + probability of B– probability of events when A and B happen together.

Cor I: When A and B are mutually exclusive events, then $P(A \cap B) = 0$ (since both cannot happen at the same time).

In this case, $P(A \cup B) = P(A) + P(B)$

Let us consider an example to explain this concept. A card is drawn from a well shuffled pack of 52 cards. What is the probability that the drawn card is either queen or heart?

Two events are described in this question. 1st event that the drawn card is queen is described by A and the other that the card drawn is heart is described by B.

$$P(A) = \frac{4}{52} = \frac{\text{number of queen}}{\text{total number of cards}}$$

$$P(B) = \frac{\text{number of hearts}}{\text{total number of cards}} = \frac{13}{52}$$

$$P(A \cap B) = \frac{\text{number of queen of hearts}}{\text{total number of cards}} = \frac{1}{52}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{4}{52} + \frac{13}{52} - \frac{1}{52} = \frac{16}{52} = \frac{4}{13}$$

Consider another example for more clarification

Q. A box contain 150 Nuts and 50 Bolts half of each type are rusted. What is the probability of one item drawn from the box to be rusted or bolt?

Sol: Here total number of items = 150 + 50 = 200

So, any experiment will be out of 200 items.

Hence, *n(S)* = 200

$$P(A) = \text{probability of rusted item} = \frac{100}{200}$$

(Since half of them are rusted, that is out of 200 items 100 items are rusted.)

$$P(B) = \text{probability of bolt} = \frac{50}{200}$$

Since out of 200 items, 50 items are bolt.

$$P(A \cap B) = \text{probability of rusted and bolt} = \frac{25}{200}$$

(Since out of 200 items in box 25 bolt are rusted).

Now, probability of rusted or bolt is

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{100}{200} + \frac{50}{200} - \frac{25}{200} = \frac{125}{200} = \frac{5}{8}$$

We had discussed the probability of an outcome when one item was drawn out of a given number of items. Now, let us consider when more than one items is drawn from the given items, then how the probability of an event is described. For example, when 2 cards are drawn from a well shuffled pack of 52 cards. What is the probability that both cards are Ace?

Here, the total ways of drawing 2 cards out of 52 cards can be in ${}^{52}C_2$ ways. Hence, the number of all outcomes must be ${}^{52}C_2$ $\therefore$ *n(S)*= ${}^{52}C_2$

Since there are four aces available in the pack. So, out of 4 cards 2 cards can be choosen in ${}^{4}C_2$ ways.

Hence favourable events *n(F)* = ${}^{4}C_2$

Hence required probability = $\frac{{}^{4}C_2}{{}^{52}C_2}$

Consider another example for discussion

A bag contain 6 red and 8 black balls, 4 balls are drawn at random from the bag. What is the probability

that 2 of them are red and 2 black?
Here, total number of ball = 6 + 8 = 14
Out of 14 balls, 4 balls can be drawn in ${}^{14}C_4$ ways

$\therefore n(S) = {}^{14}C_4$

Since, in our favour, we like 2 black and 2 red balls. Hence, the way of selecting 2 red ball out of 6 red balls is ${}^{6}C_2$ ways and 2 black balls out of 8 black balls is ${}^{8}C_2$ ways.
Hence, the total ways of selection
$= {}^{6}C_2 \times {}^{8}C_2$ i.e. $n(F) = {}^{6}C_2 \times {}^{8}C_2$

Required probability $= \frac{{}^{6}C_2 \times {}^{8}C_2}{{}^{14}C_4}$

Successive drawing

From a well shuffled pack of 52 cards, 4 cards are drawn. Find the probability of getting all heart, if they are drawn

i) at random
ii) one by one with replacement
iii) one by one without replacement

In random experiment, all cards are drawn at a time with out any pre occupied notions.

i) In the first case 4 cards can be drawn out of 52 cards in ${}^{52}C_4$ ways
$\therefore n(S) = {}^{52}C_4$
In our favour, 4 cards is required out of 13 hearts cards in ${}^{13}C_4$ ways.

$\therefore$ Required probability $= \frac{{}^{13}C_4}{{}^{52}C_4}$

$= \frac{13!}{4!9!} \times \frac{4!48!}{52!} = \frac{13\times12\times11\times10}{52\times51\times50\times49} = \frac{11}{4165}$

ii) Since, four card drawn one by one with replacement, then probability of drawing cards each time remains same. Hence probability of drawing an heart from pack $= \frac{13}{52} = \frac{1}{4}$ which repeat four times.

Hence required probability $= \frac{1}{4}\times\frac{1}{4}\times\frac{1}{4}\times\frac{1}{4} = \frac{1}{256}$

iii) When cards are drawn one by one without replacement. Then in each time cards is reducing by 1.

First event of drawing a heart has probability $= \frac{13}{52}$

Now, in second time, the total number of cards gets reduced by 1 as one heart is already drawn from the pack. In second attempt drawing a heart card out of 12 cards in remaining 51 cards.

Hence, probability of drawing a heart in second attempt $= \frac{12}{51}$

Similarly, probability of drawing a heart in the 3rd attempt $= \frac{11}{50}$ and in 4th attempt $= \frac{10}{49}$

$\therefore$ Since, we want all four heart. Hence required probability $= \frac{13}{52}\times\frac{12}{51}\times\frac{11}{50}\times\frac{10}{49} = \frac{11}{4165}$

Independent events

Two events E_1 and E_2 are said to be independent, if the occurence of the event E_2 is not affected by the occurence or non-occurence of the event E_1. For example, if two unbiased dice are rolled, the outcome of each of the dice will be independent of the outcome of the other dice.

If A and B are two independent events, then

$$P(A\cap B) = P(A)\times P(B)$$

If A, B and C are any three independent events, then

$$P(A\cap B\cap C) = P(A)\times P(B)\times P(C)$$

Complementary events

If A be an event with probability P(A). Then complementary event of A is denoted by $P(\overline{A})$ or P(A) is such an event whose impression is not A. Therefore, $P(A) + P(\overline{A}) = 1$

Cor I. If A and B are independent events, then their complementary events are also independent.

For example, a problem is given to A and B for solving. Probability that A can solve the problem is $\frac{1}{3}$ and that B can solve is $\frac{2}{5}$. Then find the probability that both of them solved.

Since solving of problems are independent by nature as solving or not solving of one does not interfere with the other's solving or not solving, we have to find the probability that both of them solved the problem.

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$P(A \cap B) = P(A) \times P(B) = \frac{1}{3} \times \frac{2}{5} = \frac{2}{15}$

If we have to find the probability that exactly one of them solve the problem. Then we can think that there are two cases possible either A solve and B not solve or A not solve and B solve the problem and we can proceed as

$P(A) = \frac{1}{3} \quad \therefore P(\overline{A}) = 1 - P(A) = 1 - \frac{1}{3} = \frac{2}{3}$

$P(B) = \frac{2}{5} \quad \therefore P(\overline{B}) = 1 - P(B) = 1 - \frac{2}{5} = \frac{3}{5}$

$\therefore$ Required probability $P(A \cap \overline{B})$ or $P(\overline{A} \cap B)$

$P(A) \times P(\overline{B}) + P(\overline{A}) \times P(B)$

$= \frac{1}{3} \times \frac{3}{5} + \frac{2}{3} \times \frac{2}{5} = \frac{3}{15} + \frac{4}{15} = \frac{7}{15}$

Consider some more examples for discussion

1. A policeman fires 4 bullets on a target. The probability of hitting a target by a bullet is $\frac{2}{3}$. What is the probability that
i) he hit the target all times
ii) he does not hit the target
iii) he hit the target

i) Here probability of hitting a target $= \frac{2}{3}$

$\therefore$ In 4 trials probability of hitting a target $= \left(\frac{2}{3}\right)^4$

ii) Probability that he can not hit target in a single trial $= 1 - \frac{2}{3} = \frac{1}{3}$

When he does not hit target in all 4 trails.

Then probability $= \left(\frac{1}{3}\right)^4$

iii) When we have to find probability of hitting the target, it means that we have to calculate hitting of one time, two times, three times and four times. But, it will be a convenient and better method to find the probability of not hitting all four times and then subtract it from 1.

$\therefore$ Probability of hitting + probability of not hitting= 1

$\therefore$ Probability that he hit the target =1- probability that he does not hit the target.

$= 1 - \left(\frac{1}{3}\right)^4 = \frac{80}{81}$

Baye's theorem

Let S be sample space in which a set of event {F_1, F_2, F_3 .... E_n}be described from a portion of an event. If A be any event then

$$P\left(\frac{E_k}{A}\right) = \frac{P(E_k)P(A/E_k)}{\sum_{i=1}^{n} P(A_1)P(A/A_1)} \quad \text{Where } 1 \le k \le n$$

Explanations

There are three bags, each containing a number of red and white balls. If a ball is drawn from any of the bag and is found that it is red in colour. Then what is the probability that it is drawn from first bag.

Since the red ball could be drawn from any of the bags and the probability of drawing of red ball from an individual bag depends upon two factors. One, selection of bag itself and secondly number of red balls out of total balls put in an individual bag. Finally, selection of balls depends upon the selection of bag. Hence conditional probability is applied for each case.

Consider one of example to solve a question based on baye's theorem.

There are three bags A, B and C in a cabin. The probability of their selection by an individual are $\frac{2}{3}, \frac{1}{6}$ and $\frac{1}{6}$ respectively. If bag A contain 4 red and 3 white balls, bag B contain 3 red and 2 white, while bag C contain 2 red and 4 white. If a bag is selected at random and a ball is drawn from the bag and is found to be red.

Then find the respective probability that it is drawn from A, B and C.

Solution: Since the drawing of red balls depend upon the selection of bag, conditional probability can be applied.

It it given that the selection of bags has probabilities $\frac{2}{3}, \frac{1}{6}, \frac{1}{6}$ i.e. $P(A) = \frac{2}{3}$, $P(B) = \frac{1}{6}$ and $P(C) = \frac{1}{6}$

Now, let E be the event of drawing a red ball from selected bag. Then $P\left(\frac{E}{A}\right)$ mean the probability of drawing a red ball from 4 red and 3 white balls. Hence probability of drawing a red ball from bag A i.e.

$$P\left(\frac{E}{A}\right) = \frac{4}{7}$$

Similarly, $P\left(\frac{E}{B}\right) = \frac{3}{5}$ and $P\left(\frac{E}{C}\right) = \frac{2}{6} = \frac{1}{3}$

Now, we have to find the probability of the ball of drawing from bag A, when it is known that red ball is drawn.

$$P\left(\frac{A}{E}\right) = \frac{P\left(\frac{E}{A}\right)P(A)}{P\left(\frac{E}{A}\right)P(A) + P\left(\frac{E}{B}\right)P(B) + P\left(\frac{E}{C}\right)P(C)}$$

$$= \frac{\frac{4}{7}\times\frac{2}{3}}{\frac{4}{7}\times\frac{2}{3}+\frac{3}{5}\times\frac{1}{6}+\frac{1}{3}\times\frac{1}{6}}$$

$$= \frac{\frac{8}{21}}{\frac{8}{21}+\frac{1}{10}+\frac{1}{18}} = \frac{8}{21}\times\frac{630}{338} = \frac{120}{169}$$

Binomial distribution

Binomial distribution is used for events when more than one trial of an event occurs. We categorise an event in only two ways, either success or failure, when there is more than one trial. Then success may be zero, one or more than one and corresponding probability of events are also different.

Let p be the probability of success in a single trial and q be the probability of failure in a single trial.

Then p + q = 1 (since in a trial there will be either success of failure). $\therefore$ q = 1 – p

Therefore, in n trials we can have any number of success between 0 to n. So, for getting r success out of n trials of an event.

$$p(X = r) = {}^{n}c_{r}\, p^{r} q^{n-r}$$

Where X denotes the number of success in n trials. If a dice is thrown 3 times and the appearance of a digit more than 4 is considered to be a success, then the probability distribution are as follows.

Here $P = \frac{2}{6} = \frac{1}{3}$ $\qquad \therefore q = 1 - p = 1 - \frac{1}{3} = \frac{2}{3}$

Now, probability of getting no success, i.e. and $n = 3$, $r = 0$

$$p(X=0) = {}^{3}C_{0}\,p^{0}q^{3-0} = 1\times\left(\frac{1}{3}\right)^{0}\times\left(\frac{2}{3}\right)^{3} = \frac{8}{27}$$

$$p(X=1) = {}^{3}C_{1}\,p^{1}q^{3-1} = 3\times\left(\frac{1}{3}\right)^{1}\times\left(\frac{2}{3}\right)^{2} = \frac{12}{27}$$

$$p(X=2) = {}^{3}C_{2}\,p^{2}q^{3-2} = 3\times\left(\frac{1}{3}\right)^{2}\times\left(\frac{2}{3}\right) = \frac{6}{27}$$

$$p(X=3) = {}^{3}C_{3}\,p^{3}q^{0} = 1\times\left(\frac{1}{3}\right)^{3}\times 1 = \frac{1}{27}$$

That is probability of getting no success, 1 success, 2 success and 3 success out of 3 trials are respectively.

$$\frac{8}{27}, \frac{12}{27}, \frac{6}{27}, \frac{1}{27}$$

One remarkable note

If there are n trials for any experiment, then $n + 1$ probability distributions are possible betwen $X = 0$ to $X = n$. But sum of all the probability is equal to 1.

i.e. $P(X = 0) + P(X = 1) + ... + P(X = n) = 1$

The probability of events E_1, E_2 .... E_n are called prior probabilities, because this event happens first and then the others happen and the probabilitiy $P(A/E)$ are called posterior probabilities, because they are determined after the results of 1st experiment.

Some important events

1. If A_1 and A_2 be two independent events with probabilities P_1 and P_2. Then the probability of an event in which A_1 occur and A_2 does not occur is $= P_1(1–P_2)$.
2. Probability that none of them occur is $= (1–P_1)\ (1–P_2).....\ (1–P_n)$
   Here, P_1, P_2 .... P_n are probability of events of A_1, A_2 .... A_n events.
3. Probability that at least one of them occur = 1-(probability that none of them occur)
   $= 1– [(1–P_1)\ (1–P_2)...\ (1–P_n)]$

Use of multinomial theorem

If a dice has m faces and an experiment is made to throw n dice of such a type at a time. Then the probability of getting sum S on its faces is given by

Coefficient of x^s in the expansion of

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$$\frac{\left(x + x^2 + x^3 + ... + x^m\right)^n}{m^n}$$ the total sample space is m^n

Poisson's distribution

When the number of trials are very large, then it is very difficult to go through binomial distribution. So, for all such cases, we can use poisson's distribution for getting the probability.

Probability of geting r success out of n experiments is given by $p(X = r)\frac{e^{\lambda}\lambda^r}{r!}$ where $\lambda = np$

Some Objective Questions

1. Two dice are tossed. The probability that the total score is a prime number is

a) $\frac{1}{6}$ b) $\frac{5}{12}$

c) $\frac{1}{2}$ d) None of these

2. The dice is tossed and it is said that either the face 1 or 2 has turned up. The probability that it is face 1 is

a) $\frac{1}{7}$ b) $\frac{4}{7}$

c) $\frac{5}{21}$ d) None of these

3. A determinant is chosen random from the set of all determinants of order 2 with elements 0 and 1 only. The probability that value of the determinant chosen is positive is

a) $\frac{1}{8}$ b) $\frac{3}{16}$

c) $\frac{1}{4}$ d) None of these

4. A bag X contains 3 white balls and 2 black balls and another bag Y contains 2 white and 4 black balls. A bag and a ball out of it are picked at random, the probability of ball being white is

a) $\frac{3}{10}$ b) $\frac{1}{6}$

c) $\frac{7}{15}$ d) None of these

5. An urn contains 10 black and 10 white balls. The probability of drawing two balls of the same colour is

a) $\frac{9}{38}$ b) $\frac{9}{19}$

c) $\frac{10}{19}$ d) None of these

6. A box contains 50 bolts and 150 nuts. Half of the bolts and half of the nuts are rusted. If one of them is chosen at random the probability that it is a bolt or it is rusted is

a) $\frac{1}{2}$ b) $\frac{5}{8}$

c) $\frac{3}{4}$ d) None of these

7. If the letters of the word 'MISSISSIPPI' are written down in a row, the probability that no two I's occur togather is

a) $\frac{1}{3}$ b) $\frac{7}{33}$

c) $\frac{6}{13}$ d) None of these

8. A and B are two independent events. The probability that both A and B occur is $\frac{1}{6}$ and the probability that neither of them occurs is $\frac{1}{3}$. The probability of occurance of A is

a) $\frac{1}{2},\frac{1}{3}$ b) $\frac{1}{3},\frac{1}{4}$

c) $\frac{1}{2},\frac{1}{4}$ d) None of these

9. If A and B are independent events, then $P(A \cap B)$ equals

a) P(A) + P(B) b) P(A)P(B)

c) P(A/B) d) P(B/A)

10. Which of the following statement is false. If M and N are any two events, the probability that exactly one of them occurs is

a) P(M) + P(N) – 2P(M $\cap$ N)

b) P(M) + P(N) – P(M $\cap$ N)

c) P(M^c) + P(N^c) + 2P($M^c \cap N^c$)

d) P(M $\cap N^c$) – P($M^c \cap$ N)

11. Let A and B be two independent events, such that P(A) = 0.4, P(B) = P and P(A $\cup$ B) = 0.7. The value of P for which A and B are independent is

a) $\frac{1}{3}$ b) $\frac{1}{4}$

c) $\frac{1}{2}$ d) $\frac{1}{5}$

12. A box contains 25 tickets numbered 1 to 25. Two tickets are drawn at random. The probability of their product being even is

a) $\frac{11}{50}$ b) $\frac{13}{50}$

c) $\frac{37}{50}$ d) None of these

13. The probability that atleast one of the events A and B occurs is 0.6. If A and B occur simultaneously with probability 0.2, then $P(\bar{A})+P(\bar{B})$ is

a) 0.4 b) 0.8

c) 1.2 d) 1.4

14. A dice is rolled 20 times. The probability of obtaining 5 for the first time at the 20th throw is

a) $\frac{1}{6}\left(\frac{5}{6}\right)^{19}$ b) $\left(\frac{5}{6}\right)\left(\frac{1}{6}\right)^{19}$

c) $\left(\frac{5}{6}\right)^{20}\left(\frac{1}{6}\right)$ d) None of these

15. If n biscuits be distributed at random among N beggers. The chance that a particular begger receives $r(<n)$ biscuits is

a) ${}^nC_r \frac{N^n}{(N-r)^{n-r}}$ b) ${}^nC_r \frac{(N-1)^{n-r}}{N^n}$

c) ${}^nC_r \frac{N-1}{N^N}$ d) None of these

16. A & B take turns in tossing a pair of dice. The first to get a thrown as sum of 7 wins. If A starts the game the chances of winning of A is

a) $\frac{6}{11}$ b) $\frac{5}{11}$

c) $\frac{1}{11}$ d) None of these

17. The probability that a 50 year old man will be alive at 60 is 0.83 and the probability that a 45 year old woman will be alive at 55 is 0.87. The probability that at least one of the will be alive ten years hence is

a) 0.221 b) 0.779

c) 0.669 d) None of these

18. Bag A contains 2 white and 3 red balls and bag B contains 4 white and 5 red balls. One ball is drawn at random from one of the bags and is found to be red, then the probability that it was drawn from the bag B is

a) $\frac{5}{9}$ b) $\frac{4}{9}$

c) $\frac{25}{52}$ d) None of these

19. If $\frac{1+3p}{3}, \frac{1-p}{4}$ and $\frac{1-2p}{2}$ are the probabilies of three mutually exclusive events, then set of all values of p is given by

a) $1 \geq p \geq -3$ b) $0 \leq p \leq \frac{1}{2}$

c) $\frac{1}{3} \leq p \leq \frac{1}{2}$ d) None of these

20. There are 6 positive and 8 negative numbers. Four numbers are chosen at random, without replacement and multiplied. The probability that the product is a positive number is

a) $\frac{500}{1001}$ b) $\frac{503}{1001}$

c) $\frac{505}{1001}$ d) None of these

21. Six dice are thrown 729 times. The number of times at least three dice show a five or six is

a) 243 b) 293

c) 423 d) None of these

22. A husband and a wife appear in a interview for

2 post in a company. Probability of their respective selection are $\frac{1}{5}$ and $\frac{1}{7}$.

Probabilities that at least one of them selected is

a) $\frac{24}{35}$ b) $\frac{11}{35}$

c) $\frac{2}{7}$ d) None of these

Solutions

1. **b.** Total score will be prime number 2, 3, 5, 7, 11 in the following 15 cases (1.1): (1.2): (2.1): (2.3): (3.2): (1.4): (4.1): (1.6): (2.5): (3.4): (4.3): (5.2): (6.1): (5.6): (6.5)

   Required probability $= \frac{\text{Favourable outcomes}}{\text{Total outcomes}}$

   $= \frac{15}{36} = \frac{5}{12}$

2. **b.** Let A be event that either 1 or 2 will turn up.

   $P(A) = \frac{2}{6} = \frac{1}{3}$

   E be the probability that 1 will turn up.

   $P(E) = \frac{1}{6}$

   $$P\left(\frac{E}{A}\right) = \frac{P(A \cap B)}{P(A)} = \frac{\frac{1}{6}}{\frac{1}{3}} = \frac{1}{6} \times \frac{3}{1} = \frac{1}{2}$$

3. **c.** Since each of the four places in a determinant of order 2 can be filled in two ways either by 0 or by 1, total number of ways is $= 2^4 = 16$.

   Further, the value of the determinant will be positive in the following 3 cases

   $\begin{matrix}1 & 1\\0 & 1\end{matrix}$ or $\begin{matrix}1 & 0\\1 & 1\end{matrix}$ or $\begin{matrix}1 & 0\\0 & 1\end{matrix}$

   Required probability $= \frac{3}{16}$

4. **b.** $\therefore$ Since there are only two bags.

   Probability of picking the bag $X = \frac{1}{2}$

   Therefore, probability of taking out a white ball from bag

   $X = \frac{1}{2} \times \frac{3}{5} = \frac{3}{10}$

   [Since bag X contain 3 white out of 5 balls].

   Similarly, probability of picking the bag $Y = \frac{1}{2}$

   Therefore, probability of taking out a white ball from bag $Y = \frac{1}{2} \times \frac{2}{6} = \frac{1}{6}$

   [Since bag Y contain only 2 ball out of 6]

   Thus the probability of taking out of a white ball out from either of the two bags (by addition law) [either by bag X or by bag Y]

   $= \frac{3}{10} + \frac{1}{6} = \frac{7}{15}$

5. **b.** 2 black balls can be drawn out of 10 in ${}^{10}C_2$ ways. Similarly 2 white balls can be drawn out of 10 in ${}^{10}C_2$ ways.

   Two balls of same colour mean probability of getting 2 black or 2 white ball.

   Required probability

   $$= \frac{{}^{10}C_2}{{}^{20}C_2} + \frac{{}^{10}C_2}{{}^{20}C_2} = 2 \times \frac{{}^{10}C_2}{{}^{20}C_2} = \frac{9}{19}$$

   Second method: The two mutual exclusive cases are

   i) both balls are white, ii) both balls are black

   i) P (both white) $= \frac{{}^{10}C_2}{{}^{20}C_2} = \frac{9}{38}$

   ii) P (both black) $\frac{{}^{10}C_2}{{}^{20}C_2} = \frac{9}{38}$

   P (both of same colour) = P (both white) + P (both black)

   $\frac{9}{38} + \frac{9}{38} = \frac{9}{19}$

6. **b.** Total number of nuts & bolts = 50 + 150=200

   Number or rusted bolts = 25

   Number of rusted nuts = 75

   P (bolt or rusted) = P (bolt) + P (rusted)– P(bolt and rusted)

P (bolt) = $\frac{50}{200}=\frac{1}{4}$

P (rusted)= $\frac{100}{200}=\frac{1}{2}$

P (bolt and rusted) = $\frac{1}{4}\times\frac{1}{2}=\frac{1}{8}$

P (bolt or rusted) = $\frac{1}{4}+\frac{1}{2}-\frac{1}{8}=\frac{5}{8}$

7. **a.** The number of permutations of the letters of the given word = $\frac{11!}{4!4!2!}$

Since number consists 4 S's and 4 I's and 2 P's.

The number of permutations in which two S's are never together = $\frac{7!}{4!2!}\ {}^{8}C_4=\frac{7!}{4!2!},\frac{8!}{4!4!}$

Required probability = $\frac{7!8!}{4!2!4!4!},\frac{4!4!2!}{11!}=\frac{7}{33}$

8. **b.** Since A and B are independent events. Hence $\overline{A}$ and $\overline{B}$ are also independent events.

Let $P(A) = p_1$ and $P(B) = p_2$

Then $P(A\cap B) = p_1p_2 = \frac{1}{6}$ .... (1)

Also $P(\overline{A}\cap\overline{B})=q_1q_2=\frac{1}{3} or \left(1-p_1\right)\left(1-p_2\right)=\frac{1}{3}$

or $1-(p_1+p_2)+p_1p_2=\frac{1}{3}$

or $p_1+p_2=\frac{5}{6}$ .... (2)

Solving (1) and (2), we get $p_1=\frac{1}{2}$ or $\frac{1}{3}$

9. **b.** If A and B are independent events, then $P(A\cap B) = P(A), P(B)$

10. **c.** The required probability = probability that M occurs N does not or N ocurs and M does not occur

$= P(M\cap N^C) + P(M^C\cap N)$

$= P(M)-P(M\cap N) + P(N)-P(M\cap N)$

$= P(M) + P(N) - 2P(M\cap N)$

11. **c.** Since A and B are independent

$P(A\cap B)=P(A)P(B)$

Also $P(A\cup B) = P(A) + P(B) - P(A\cap B)$

$\Rightarrow 0.7 = 0.4 + p - P(A)P(B)$

$\Rightarrow 0.3 = p - (0.4)p = 0.6p \Rightarrow \frac{1}{2}$

12. **c.** The product of the numbers will be even when either both of them are even or one is even and the other is odd. We have 12 even numbers and 13 odd numbers.

Number of ways of selecting 2 even numbered tickets out of 12 = ${}^{12}C_2$

Number of ways of selecting one even and one odd ticket out of 12 even and 13 odd = ${}^{12}C_1\times{}^{13}C_1$

Total number of ways of selecting 2 tickets = ${}^{25}C_3$

The required probability $=\frac{{}^{12}C_2+{}^{12}C_1\times{}^{13}C_1}{{}^{25}C_2}=\frac{37}{50}$

13. **a.** Given $P(A\cup B) = 0.6$

$P(A\cap B) = 0.2$

$P(A\cup B) = P(A) + P(B)-P(A\cap B)$

$P(A) + P(B) = 0.6 + 0.2 = 0.8$

Now $P(\overline{A})=1-P(A)$

$P(\overline{A})=1-P(B)$

Now, $P(A) + P(B) = 2-[P(A) + P(B)] = 2-0.8=1.2$

14. **d.** Total number of trials $n = 20$

Success $P=\frac{1}{6}$ and failure $q = 1-\frac{1}{6}=\frac{5}{6}$

For 1 success in 20th throws means 19 successive failure and 20th success. Hence

required probability $\left(\frac{5}{6}\right)^{19}\left(\frac{1}{6}\right)$

15. **a.** Number of biscuits = n

Number of beggers = N

One of biscuit can be distributed in N ways

$\therefore$ n biscuits can be distributed in N^n ways

Total number of ways = N^n

If a particular begger receives r biscuits, then the remaining $(n-r)$ biscuits are to be distributed among the other N–1 beggers.

This is possible in $(N-1)^{n-r}$ ways

r biscuits can be selected from n biscuits in ${}^{n}C_r$ ways.

$\therefore$ Number of favourable ways = ${}^nC_r\,(N-1)^{n-r}$

The required chance $= \dfrac{{}^nC_r(N-1)^{n-r}}{N^n}$

16. **b.** Chance of A's winning in the first throw = $\dfrac{1}{6}$.

Chance of A's winning in the second turn = chance of A's failure × chance of B's failure × chance of A's success $= \dfrac{5}{6}\times\dfrac{5}{6}\times\dfrac{1}{6}$

Similarly chance of A's winning in his third turn $\dfrac{5}{6}\times\dfrac{5}{6}\times\dfrac{5}{6}\times\dfrac{5}{6}\times\dfrac{1}{6}$ and so on

$\therefore$ A's chance of winning

$$=\frac{1}{6}+\left(\frac{5}{6}\right)2\times\frac{1}{6}+\left(\frac{5}{6}\right)4\times\frac{1}{6}+....$$

$$=\frac{\frac{1}{6}}{1-\frac{25}{36}}=\frac{6}{11}$$

17. **c.** Probability of the man living upto 60 = $P(A)$ =0.83

Probability of the man not living upto 60 $= P(\overline{A}) = 1–0.83 = 0.17$

Probabilty of the woman living upto 55=$P(B)$=0.87

Probability of the woman not living upto 55 $= P(\overline{B}) = 1– 0.87 = 0.13$

Probability that none of them lives for 10 years $= P(\overline{A}\cap\overline{B}) = 0.17\times 0.13 = 0.221$

Probability that at least one of them lives for next 10 years = 1–(none of them live upto 10 years)

$=1- P(\overline{A}\cap\overline{B}) = 1- 0.221= 0.779$

18. **c.** Let E_1 be the event that the ball is drawn from bag A, E_2 be the event that ball is drawn from bag B, and E be the event that the ball drawn is red.

We have to find $P(E_2/E)$

Since both the bags are equally likely to be selected, we have $P(E_1) = P(E_2) = \dfrac{1}{2}$

Also, $P(E/E_1) = \dfrac{3}{5}$ and $P(E/E_2)= \dfrac{5}{9}$

Hence, by Baye's theorem

$$P(E_2/E)=\frac{P(E_2)P(E/E_2)}{P(E_1)P(E/E_1)+P(E_2)P(E/E_2)}$$

$$=\frac{\frac{1}{2}\times\frac{5}{3}}{\frac{1}{2}\times\frac{3}{5}+\frac{1}{2}\times\frac{5}{9}}=\frac{25}{52}$$

19. **c.** Let A, B, C denote the mutually exclusive events, so that

$$P(A)=\frac{1+3p}{3},\ P(B)=\frac{1-p}{4},\ P(C)=\frac{1-2p}{2}$$

Since A, B, C are mutually exclusive

$\therefore 0\le P(A)+P(B)+P(C)=P(A\cap B\cap C)\le 1$

or $0\le\dfrac{1+3p}{3}+\dfrac{1-p}{4}+\dfrac{1-2p}{2}$

or $0\le 13-3p\le 12$ or $-13\le -3p\le -1$

or $13\ge 3p\ge 1$ or $1\le 3p\le 13$

or $\dfrac{1}{3}\le p\le\dfrac{13}{3}$ ....(1)

Also, $0\le P(A)\le 1\Rightarrow 0\le\dfrac{1+3p}{3}\le 1$

$\therefore 0\le 1+3p\le 3\Rightarrow -1\le 3p\le 2$

$\Rightarrow -\dfrac{1}{3}\le p\le\dfrac{2}{3}$

But $p\ge 0$, hence $0\le p\le\dfrac{2}{3}$ ....(2)

Again $0\le P(B)\le 1\Rightarrow 0\le\dfrac{1-p}{4}\le 1$

$\Rightarrow 0\le 1-p\le 4\Rightarrow -1\le -p\le 3$

$\Rightarrow 0\ge p\ge -3$ ....(3)

$\therefore 0\le p\le 1$

Similarly from $0\le P(C)\le 1$, we shall easily get

$0 \le p \le \frac{1}{2}$ ....(4)

Now value of p which will satisfy all inequalities from (1) to (4) is given by

$\frac{1}{3} \le p \le \frac{1}{2}$

20. **c.** To get the product of four numbers, as a positive number, the possible of combinations of numbers are shown below:
i) 4 positive numbers and no negative numbes
ii) 2 positive numbers and two negative numbers
iii) No positive numbers and 4 negative numbers
Numers of numbers corresponding to (i), (ii) and (iii) are $= {}^6C_4 \times {}^8C_0 + {}^6C_2 \times {}^8C_2 + {}^6C_0 \times {}^8C_4 = 505$
Out of 14 numbers 4 can be selected in

$${}^{14}C_4 = \frac{14\times13\times12\times11}{1\times2\times3\times4} = 1001$$

Required probability $= \frac{505}{1001}$

21. **b.** Favourable event {5, 6}
$\therefore n(F) = 2, n(S) = 6$

$$p = \frac{2}{6} = \frac{1}{3}, q = \frac{2}{3}$$

Required number

$$= 729\left[{}^6C_3\left(\frac{1}{3}\right)^3\left(\frac{2}{3}\right)^3 + {}^6C_4\left(\frac{1}{3}\right)^4\left(\frac{2}{3}\right)^2 + {}^6C_5\left(\frac{1}{3}\right)^5\left(\frac{2}{3}\right) + {}^6C_6\left(\frac{1}{3}\right)^6\right]$$

$$= \left[\frac{6\times5\times4}{1\times2\times3}\times8 + \frac{6\times5}{1\times2}\times4 + 6\times2 + 1\right]$$

$= [160 + 60 + 12 + 1] = 293$

22. **b.** Probability that at least of them select
$= 1–$ P(none of them select)

$= 1 - P(\bar{A} \cap \bar{B})$

$$= 1 - \left[P(\bar{A}) \times P(\bar{B})\right] = 1 - \frac{4}{5}\times\frac{6}{7} = 1 - \frac{24}{35} = \frac{11}{35}$$

1. In a lottery of 50 tickets numbered 1 to 50, two tickets are drawn simultaneously. Compute the probability that both tickets have prime numbers?

a) $\frac{3}{25}$ b) $\frac{3}{35}$

c) $\frac{2}{35}$ d) None

2. A committee of 5 is to be selected at random from a group of 10 men and 8 women. What is the probability that there will be exactly 3 men in the committee

a) $\frac{20}{51}$ b) $\frac{1}{3}$

c) $\frac{5}{18}$ d) $\frac{3}{10}$

3. A bag contains 4 red and 5 white balls. Two balls are drawn at random. Then the probability that the both the balls are white is

a) $\frac{2}{5}$ b) $\frac{5}{9}$

c) $\frac{5}{18}$ d) $\frac{3}{10}$

4. The probability that a non-leap year selected at random contains 53 Sundays is

a) $\frac{1}{7}$ b) $\frac{2}{7}$

c) $\frac{3}{7}$ d) $\frac{4}{7}$

5. The probability of obtaining only 5 head in tossing of 6 coins simultaneously is

a) $\frac{3}{64}$ b) $\frac{6}{35}$

c) $\frac{3}{32}$ d) $\frac{5}{6}$

6. If the probability that a research project will be well planned is 0.80 and the probability that it will be well executed is 0.72. What is the probability that a research project that is well planed will also be well executed?

a) $\frac{9}{10}$ b) $\frac{10}{11}$

c) $\frac{7}{100}$ d) $\frac{72}{125}$

7. In a class there are 20 students, out of which 8 are girls, 10 students are intelligent, 12 are rich. If a student is selected a random. What is the probability that it is rich and intelligent boy?

a) $\frac{3}{25}$ b) $\frac{9}{100}$

c) $\frac{9}{50}$ d) $\frac{9}{25}$

8. In an interview for two posts, the probability for husband's selection is $\frac{1}{5}$ and that of the wife's selection is $\frac{1}{7}$. The probability that one of them selected is

a) $\frac{2}{7}$ b) $\frac{5}{7}$

c) $\frac{1}{35}$ d) $\frac{11}{35}$

9. Reema speaks 60% truth while Seema speaks 70% truth. While they are narrating the same story, what is the probability that they will contradict each other.

a) 28% b) 46%

c) 52% d) 54%

10. A bag contain 250 nuts and 150 bolts. Half of the nuts and half of the bolts are rusted. An item is drawn at random. What is the probability that it is rusted bolt.

a) $\frac{3}{16}$ b) $\frac{2}{15}$

c) $\frac{5}{8}$ d) $\frac{5}{16}$

11. A bag contain 5 white and 3 black balls. Two balls are drawn at random. What is the probability that both balls are of different colour.

a) $\frac{1}{6}$ b) $\frac{2}{7}$

c) $\frac{1}{7}$ d) None

12. From a group of 13 scientists which contain 5 Mathematicians and 8 physicists, it is required to appoint a committee of two. If the selection is made without knowing the identity if the scientists, what is the probability that one will be a mathematician and other a physicist?

a) $\frac{20}{39}$ b) $\frac{10}{39}$

c) $\frac{5}{39}$ d) None

13. Of a total of 600 bolts, 20% are too large and 10% too small. The remainder are considered to be suitable. If a bolt is selected at random,. What is the probability that it will be suitable?

a) $\frac{1}{2}$ b) $\frac{3}{10}$

c) $\frac{7}{10}$ d) None

14. A problem is given to Suresh and Mahesh for solution. The probability that Suresh will solve the problem is $\frac{2}{5}$ and that Mahesh will solve is $\frac{4}{5}$. What is the probability that the problem is not solved.

a) $\frac{3}{5}$ b) $\frac{3}{25}$

c) $\frac{8}{25}$ d) $\frac{12}{25}$

15. A money bag has two compartments. The first compartment contain five 5 rupees notes and ten 10 rupees notes. The second compartment contain ten 5 rupees notes and Five 10 rupees notes. One note from the bag is drawn at random. What is the probability that it is a 5 rupees note?

a) $\frac{1}{4}$ b) $\frac{1}{3}$

c) $\frac{5}{6}$ d) $\frac{1}{2}$

16. From a well shuffled pack of 52 cards, a card is drawn. What is the probability that it is a black queen or a king.

a) $\frac{1}{26}$ b) $\frac{1}{13}$

c) $\frac{3}{26}$ d) None

17. A police man fires 4 bullets to shootout criminals. The probability that he hit by using the four bullets are $\frac{1}{3},\frac{1}{4},\frac{1}{5},\frac{1}{6}$ respectively. The probability that criminal is still alive is

a) $\frac{1}{2}$ b) $\frac{1}{4}$

c) $\frac{1}{360}$ d) $\frac{1}{3}$

18. Three coins are tossed together. What is the probability that the sum of digits appearing on its top is 10 is

a) $\frac{3}{8}$ b) $\frac{1}{2}$

c) $\frac{5}{8}$ d) $\frac{2}{8}$

19. A pair of dice thrown together. Then the probability that the sum of digits appear on its top is 10 is

a) $\frac{7}{12}$ b) $\frac{1}{6}$

c) $\frac{1}{12}$ d) $\frac{1}{9}$

20. Ratika has tossed a pair of dice together and she got the sum of the digits as 10. Then the probability that 4 has appear on one of the dice is

a) $\frac{2}{3}$ b) $\frac{1}{3}$

c) $\frac{1}{12}$ d) $\frac{1}{6}$

21. From a well shuffled pack of 52 cards, a card is drawn at random what is the probability that it is a heart or an ace?

a) $\frac{3}{13}$ b) $\frac{17}{52}$

c) $\frac{4}{13}$ d) None

22. If one word out of the total words which can be made with or without meaning from the letters of the word 'DELHI' is picked up at random. What is probability that vowels are together.

a) $\frac{1}{5}$ b) $\frac{2}{5}$

c) $\frac{3}{5}$ d) $\frac{1}{120}$

23. Two cards accidently dropped from a well shuffled pack of 52 cards. What is the probability that both are king?

a) $\frac{1}{13}$ b) $\frac{1}{17}$

c) $\frac{4}{52 \times 51}$ d) $\frac{1}{221}$

24. What is the probability that a leap year contain 53 Mondays?

a) $\frac{1}{7}$ b) $\frac{53}{365}$

c) $\frac{7}{365}$ d) $\frac{2}{7}$

25. If an average of 1 electric bulb out of 10 are defective, Find the probability that out of 5 bulbs selected at random, 4 of them are defective?

a) $\frac{4}{10}$ b) $\frac{1}{(10)^4}$

c) $\frac{5}{(10)^4}$ d) $\frac{9}{2 \times (10)^4}$

Answers

1. b	4. a	7. c	10. a	13. c	16. c	19. c	22. b	25. d
2. a	5. c	8. a	11. c	14. b	17. d	20. a	23. d	
3. c	6. d	9. b	12. a	15. d	18. a	21. c	24. d	