Solution Of Self Evaluation Test-02 (Paper-I)

Topics :Transformation Formula & Trigonometric Equation)

SECTION - I

Straight Objective Type

This section contains 8 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

29. The minimum value of $(\sin\theta + \operatorname{cosec}\theta)^2 + (\cos\theta + \sec\theta)^2$ is

(A) 7 (B) 10 (C) 9 (D) 11

Soln. $\sin^2\theta + \operatorname{cosec}^2\theta + 2 + \cos^2\theta + \sec^2\theta + 2$

$= 4 + (\sin^2\theta + \cos^2\theta) + (\operatorname{cosec}^2\theta + \sec^2\theta)$

$= 4 + 1 + \frac{1}{\sin^2\theta} + \frac{1}{\cos^2\theta} = 5 + \frac{\cos^2\theta + \sin^2\theta}{\sin^2\theta\cos^2\theta}$

$= 5 + \frac{1 \times 4}{4\sin^2\theta\cos^2\theta} = 5 + \frac{4}{(2\sin\theta\cos\theta)^2} = 5 + \frac{4}{\sin^2 2\theta}$

For minimum value $\sin^2 2\theta$ should be maximum

$\therefore$ max value of $\sin^2 2\theta = 1$

$\therefore$ minimum value of the expression $= 5 + 4 = 9$

30. The number of real solutions of the equation $\cos(e^x) = 5^x + 5^{-x}$ is

(A) 0 (B) 1 (C) 2 (D) 3

Soln. L.H.S of the equation lies between -1 to 1. while RHS is greater than equal to 2 as sum of positive number and its reciprocal is always greater than or equal to 2. Hence, equation has no. soln. The number of solution is 0

Ans (A)

31. If $2\sin\alpha\cos\beta\sin\gamma = \sin\beta\sin(\alpha+\gamma)$, where $\alpha, \beta, \gamma \neq n\pi$, $n \in Z$, then $\cot\alpha$, $\cot\beta$, $\cot\gamma$ are in

(A) A P (B) G.P (C) H.P (D) A.G.P

Sol n $2\sin\alpha\cos\beta\sin\gamma = \sin\beta\sin(\alpha+\gamma)$

$\Rightarrow 2\sin\alpha\cos\beta\sin\gamma = \sin\beta(\sin\alpha\cos\gamma + \cos\alpha\sin\gamma)$

$\Rightarrow 2\sin\alpha\cos\beta\sin\gamma = \sin\alpha\sin\beta\cos\gamma + \sin\beta\sin\gamma\cos\alpha$

Divide by $\sin\alpha\sin\beta\sin\gamma$ throughout

$\Rightarrow 2\cot\beta = \cot\alpha + \cot\gamma$

$\therefore$ $\cot\alpha$, $\cot\beta$, $\cot\gamma$ are in AP Ans (A).

32. If $k_1 = \tan 81\theta - \tan\theta$ and $k_2 = \dfrac{\sin\theta}{\cos 3\theta} + \dfrac{\sin 3\theta}{\cos 9\theta} + \dfrac{\sin 9\theta}{\cos 27\theta} + \dfrac{\sin 27\theta}{\cos 81\theta}$, then a relation between k_1 and k_2 is

(A) $k_1 = 2k_2$ (B) $k_1 = k_2$ (C) $k_1 = 3k_2$ (D) None.

Sol n We have $\dfrac{\sin\theta}{\cos 3\theta} = \dfrac{2\sin\theta\cos\theta}{2\cos 3\theta\cos\theta} = \dfrac{\sin 2\theta}{2\cos 3\theta\cos\theta}$

$$= \frac{1}{2}\left[\frac{\sin(3\theta-\theta)}{\cos 3\theta\cos\theta}\right]$$

$$= \frac{1}{2}\left[\frac{\sin 3\theta\cancel{\cos\theta}}{\cos 3\theta\cancel{\cos\theta}} - \frac{\cancel{\cos 3\theta}\sin\theta}{\cancel{\cos 3\theta}\cos\theta}\right]$$

i.e $\dfrac{\sin\theta}{\cos 3\theta} = \dfrac{1}{2}[\tan 3\theta - \tan\theta]$

Similarly, $\dfrac{\sin 3\theta}{\cos 9\theta} = \dfrac{1}{2}[\tan 9\theta - \tan 3\theta]$

$\dfrac{\sin 9\theta}{\cos 27\theta} = \dfrac{1}{2}[\tan 27\theta - \tan 9\theta]$

$\dfrac{\sin 27\theta}{\cos 81\theta} = \dfrac{1}{2}[\tan 81\theta - \tan 27\theta]$

Adding all we get, $K_2 = \frac{1}{2}[\tan 81\theta - \tan\theta)$

$= \frac{1}{2} k_1$

$\therefore \boxed{2k_2 = k_1}$ Ans (A)

33. The value of $\tan A + 2\tan 2A + 2^2 \tan^2 2A + 2^3 \tan 2^3 A + 2^4 \tan 2^4 A + 32\cot 32A$ is equal to.

(A) $\cot A + \tan A$ (B) $\cot A$

(C) $\tan A$ (D) $\cot A - \tan A$

Sol^n $\because \cot A - \tan A = \cot A - \frac{1}{\cot A} = \frac{\cot^2 A - 1}{\cot A} = 2\cot 2A$

Similarly, $2\cot 2A - 2\tan 2A = 2\left(\cot 2A - \frac{1}{\cot 2A}\right)$

$= 2 \cdot \frac{\cot^2 2A - 1}{\cot 2A} = 2^2 \cot 4A$ etc.

Now, $\cot A - (\cot A - \tan A - 2\tan 2A - 4\tan 4A - 8\tan 8A - 16\tan 16A - 32\cot 32A)$

$= \cot A - (2\cot 2A - 2\tan 2A - 4\tan 4A - 8\tan 8A - 16\tan 16A - 32\cot 32A)$

$= \cot A - (4\cot 4A - 4\tan 4A - 8\tan 8A - 16\tan 16A - 32\cot 32A)$

$= \cot A - (8\cot 8A - 8\tan 8A - 16\tan 16A - 32\cot 32A)$

$= \cot A - (16\cot 16A - 16\tan 16A - 32\cot 32A)$

$= \cot A - (32\cancel{\cot 32A} - 32\cancel{\cot 32A}) = \cot A$ Ans

34. The value(s) of k for which the equation $\sin x + \cos x = \min\limits_{p \in R}\{1, p^2 - 4p + k\}$ has the solution $n\pi + (-1)^n \frac{\pi}{4} - \frac{\pi}{4}$, $n \in I$ is / are

(A) $k > 5$ (B) $k = 2$ (C) $k_1 = 3k_2$ (D) None.

Soln.

$$p^2 - 4p + k = p^2 - 4p + 4 + k - 4$$
$$= (p-2)^2 + k - 4$$

If $\sin x + \cos x = 1 \Rightarrow \frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x = \frac{1}{\sqrt{2}}$

$\Rightarrow \sin x \cos\frac{\pi}{4} + \cos x \sin\frac{\pi}{4} = \sin\frac{\pi}{4}$

$\Rightarrow \sin\left(x + \frac{\pi}{4}\right) = \sin\frac{\pi}{4}$

$\left(x + \frac{\pi}{4}\right) = n\pi + (-1)^n \frac{\pi}{4}, n \in I$

$\therefore x = n\pi + (-1)^n \frac{\pi}{4} - \frac{\pi}{4}, n \in I$

This is possible if $\min\{1, p^2 - 4p + k\} = 1$

i.e $\min\{1, (p-2)^2 + k - 4\} = 1$

$\Rightarrow (p-2)^2 + k - 4 > 1 \Rightarrow k - 4 > 1$

$\boxed{k > 5}$ Ans.

as min of $(p-2)^2 = 0$

35. The sides of a triangle are as $1 : \sqrt{3} : 2$. Then the value of $\frac{q}{2p}$ for which the ratio of corresponding angles to one another is $p : q : 3$ is

(A) $\frac{2}{3}$ (B) $\frac{1}{3}$ (C) 2 (D) 1

Soln. We know by sine formula $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$\therefore \sin A : \sin B : \sin C = a : b : c$

$= 1 : \sqrt{3} : 2$

$= \frac{1}{2} : \frac{\sqrt{3}}{2} : 1$

$\therefore A = 30^\circ, \quad B = 60^\circ, \quad C = 90^\circ$

Ratio of angles are $30 : 60 : 90$ i.e $1 : 2 : 3$

But given ratio are $p : q : 3$ $\therefore p = 1, q = 2$

$\therefore \frac{q}{2p} = 1$ Ans (D)

36. For $x \in \left[0, \frac{k\pi}{2}\right]$, $k \in N$, $2\tan^2 x - 5\sec x$ is equal to 1 for exactly 7 distinct value of x, then greatest value of k is

(A) 12 (B) 15 (C) 9 (D) 6

Soln. $2\tan^2 x - 5\sec x = 1$

$\Rightarrow 2\sec^2 x - 2 - 5\sec x = 1 \Rightarrow 2\sec^2 x - 5\sec x - 3 = 0$

$\Rightarrow (2\sec x + 1)(\sec x - 3) = 0$ But $2\sec x + 1 \neq 0$

$\Rightarrow \sec x = 3$. which will give two values in $[0, 2\pi]$

four " " $[0, 4\pi]$

six " $[0, 6\pi]$

seventh value lies in 1st quadrant $\left(6\pi, 6\pi + \frac{\pi}{2}\right)$

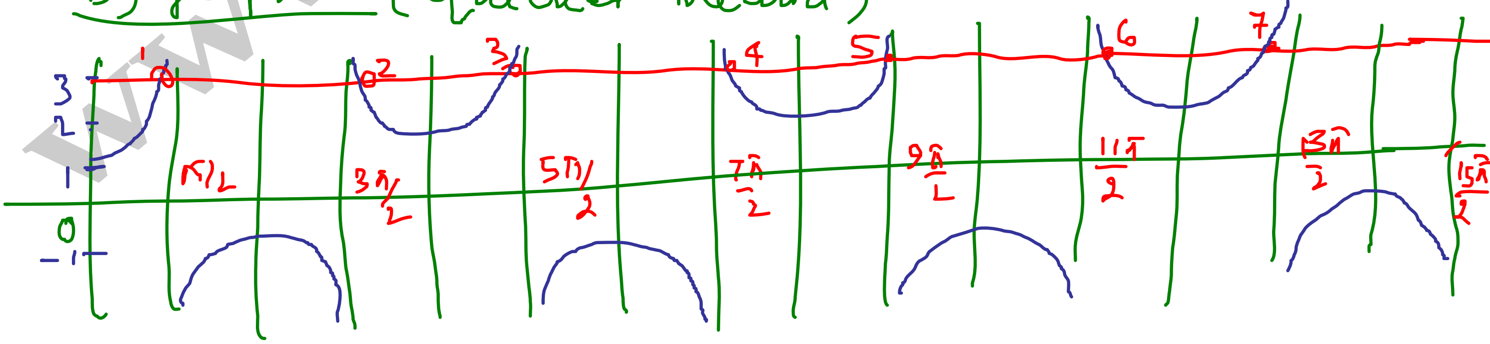
to $\left(6\pi, 6\pi + \frac{3\pi}{2}\right)$

$\therefore$ Seven value can be obtained in interval

$\left(0, \frac{13\pi}{2}\right)$ to $\left(0, \frac{15\pi}{2}\right)$

$\therefore$ K can be 13, 14, 15 Ans (15)

By graphs - (quicker method)



Clearly, for 7 Solution interval should be min $\left(0, \frac{13\pi}{2}\right)$ and max. $\left(0, \frac{15\pi}{2}\right)$.

$\therefore$ value of k lies between 13 to 15

max. value 15. Ans.

SECTION - II

Multiple Correct Answer Type

This section contains 5 multiple correct answer(s) type questions. Each question has 4 choices (A), (B), (C) and (D), out of which **ONE OR MORE** is/are correct.

37. $\tan\alpha = \sqrt{a}$, where a is a rational number and is not a perfect square. Then which of the following is an irrational number?

(A) $\sin 2\alpha$ (B) $\operatorname{cosec} 2\alpha$ (C) $\cos 2\alpha$ (D) $\tan 2\alpha$

Soln
A, B, D

$\sin 2\alpha = \dfrac{2\tan\alpha}{1+\tan^2\alpha} = \dfrac{2\sqrt{a}}{1+a}$ is irrational no

$\therefore \operatorname{cosec} 2\alpha$ is also an irrational number.

$\cos 2\alpha = \dfrac{1-\tan^2\alpha}{1+\tan^2\alpha} = \dfrac{1-a}{1+a}$ is rational number

$\tan 2\alpha = \dfrac{2\tan\alpha}{1-\tan^2\alpha} = \dfrac{2\sqrt{a}}{1-a}$ is an irrational number.

38. If $\cos 5\theta = a\cos^5\theta + b\cos^3\theta + c\cos\theta$, Then c can not be equal to

(A) 5 (B) 4 (C) 3 (D) 2

Soln: $\cos 5\theta = \cos(3\theta + 2\theta)$

$= \cos 3\theta \cos 2\theta - \sin 3\theta \sin 2\theta$

$\therefore \cos 5\theta = (4\cos^3\theta - 3\cos\theta)(2\cos^2\theta - 1) - (3\sin\theta - 4\sin^3\theta)2\sin\theta\cos\theta$

$= 8\cos^5\theta - 10\cos^3\theta + 3\cos\theta - 2(3-4\sin^2\theta)\sin^2\theta\cos\theta$

$= 8\cos^5\theta - 10\cos^3\theta + 3\cos\theta - 2(4\cos^2\theta - 1)(1-\cos^2\theta)\cos\theta$

$= 8\cos^5\theta - 10\cos^3\theta + 3\cos\theta + 8\cos^5\theta - 10\cos^3\theta + 2\cos\theta$

$= 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$

On comparing $a = 16$, $b = -20$, $c = 5$ $\therefore$ Ans. B, C, D

39. If $f(x) = \dfrac{\sin 3x}{\sin x}$, $x \neq n\pi$. Then

(A) $\max(f) < 3$ (B) $\min(f) = -1$

(C) Range $(f) \in [-1, 3)$ (D) Domain$(f) = R - n\pi, n \in Z$

Soln. $\because$ $f(x) = \dfrac{\sin 3x}{\sin x} = \dfrac{3\sin x - 4\sin^3 x}{\sin x}$

$= 3 - 4\sin^2 x$.

$\because x \neq n\pi, n \in Z$, $\therefore$ $0 < \sin^2 x \leq 1 \Rightarrow 0 < 4\sin^2 x \leq 4$

$\Rightarrow -4 \leq -4\sin^2 x < 0 \Rightarrow -1 \leq 3 - 4\sin^2 x < 3$

$\therefore$ Max. $(f) < 3$ $\therefore$ A is correct

min $(f) = -1$ B is correct

Range $[-1, 3)$ C is correct

domain $R - n\pi$, $n \in Z$ $\therefore$ D is correct.

$\therefore$ A, B, C, D all are Correct.

40. The equation $\sec\theta + \operatorname{cosec}\theta = k$ has

(A) Two roots $\in [0, 2\pi]$ if $k^2 < 8$

(B) Four roots $\in [0, 2\pi]$ if $k^2 > 8$

(C) No roots $\in [0, 2\pi]$ if $k^2 > 8$

(D) Four roots $\in [0, 2\pi]$ if $k^2 < 8$

Soln. $\sec\theta + \operatorname{cosec}\theta = k$

$$k^2 = \sec^2\theta + \operatorname{cosec}^2\theta + 2\sec\theta\operatorname{cosec}\theta$$

$$= \frac{1}{\cos^2\theta} + \frac{1}{\sin^2\theta} + \frac{2}{\sin\theta\cos\theta}$$

$$= \frac{1 \times 4}{4 \times \sin^2\theta\cos^2\theta} + \frac{2 \times 2}{2\sin\theta\cos\theta} = \frac{4}{\sin^2 2\theta} + \frac{4}{\sin 2\theta}$$

$\therefore$ min value of $k^2 = \frac{4}{1^2} + \frac{4}{1} = 8$

when $\sin 2\theta = 1 \Rightarrow 2\theta = \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2} \ldots$

$\Rightarrow \theta = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4} \ldots$

$\therefore$ But, $\frac{\pi}{4}, \frac{5\pi}{4} \in [0, 2\pi]$

when $k^2 < 8$, there are two roots in $[0, 2\pi]$

and when $k^2 > 8$, " no root in $[0, 2\pi]$

A and C are correct answer.

41. Given that $\sin x = |\sqrt{3} + \cos x| \;\forall\; x \in [0, 3\pi]$.

Then

(A) The given equality is not valid for any x.

(B) The number of solution is 4 in given interval

(C) The equality holds for $x = n\pi + (-1)^n \frac{\pi}{3} + \frac{\pi}{6}, n \in I$

(D) Total number of solution is 3.

Soln. $\because -1 \le \cos x \le 1 \quad \therefore \sqrt{3} + \cos x > 0$

$\therefore |\sqrt{3} + \cos x| = \sqrt{3} + \cos x$

$\therefore \sin x = \sqrt{3} + \cos x$

$\sqrt{3} - 1 \le \sqrt{3} + \cos x \le \sqrt{3} + 1$

$\therefore \sqrt{3} - 1 \le \sin x \le 1 \quad [\because \text{Max. of } \sin x = 1] - (1)$

Also, $\sin x - \cos x = \sqrt{3}$.

Squaring, $1 - \sin 2x = 3 \Rightarrow \sin 2x = -2$

which will not hold for any value of x

SECTION-III

Linked Comprehension Type

This section contains 2 paragraph. Based upon the first paragraph two multiple choice questions have to be answered and based upon the first paragraph three multiple choice questions have to be answered. . Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONLY ONE** is correct.

Paragraph for Question No's 42 to 43.

We know that if α and β be the roots of $ax^2+bx+c=0$ $a\neq 0$, then $\alpha+\beta=-\frac{b}{a}$, $\alpha\beta=\frac{c}{a}$

If $\tan A$ and $\tan B$ be the roots of quadratic equation $abx^2-c^2x+ab=0$, where a, b, c are the sides of triangle. Then we can find the corresponding trigonometric values by using sine, cosine and tangent based formulae of a triangle. On the basis of the above information answer the following.

42. The value of $\sin^2 A+\sin^2 B+\sin^2 C$ equals

(A) 2 (B) $\frac{3}{2}$ (C) $\frac{3\sqrt{3}}{2}$ (D) $\sqrt{2}+1$

Sol^n. $\tan A+\tan B=\frac{c^2}{ab}$ and $\tan A\tan B=1$

$\Rightarrow \tan A=\cot B$

$\Rightarrow A=B=\pi/4$

$\therefore C=\pi/2$

$\therefore \sin^2 A+\sin^2 B+\sin^2 C$

$=\left(\frac{1}{\sqrt{2}}\right)^2+\left(\frac{1}{\sqrt{2}}\right)^2+(1)^2=\frac{1}{2}+\frac{1}{2}+1=2$ Ans. (A)

43. $\tan^2 A+\tan^2 B+\cos^2 C$ equals.

(A) $\frac{c^4-2a^2c^2}{a^2b^2}$ (B) $\frac{c^2(a^2-b^2)}{a^2b^2}$

(C) $\frac{c^4-2a^2b^2}{a^2b^2}$ (D) $\frac{c^2+a^2b^2}{a^2b^2}$

Soln. we have $\tan^2 A + \tan^2 B + \cos^2 C$

$= (\tan A + \tan B)^2 - 2\tan A \tan B + \cos^2 \pi/2$

$= \left(\frac{c^2}{ab}\right)^2 - 2 \times 1 + 0 \Rightarrow \frac{c^4}{a^2b^2} - 2$

$= \frac{c^4 - 2a^2b^2}{a^2b^2}$ Ans. (C)

Paragraph for Question No's 44 to 46.

Let us consider the sum of series given below

$\sin\alpha + \sin(\alpha+\beta) + \sin(\alpha+2\beta) + \cdots + \sin[\alpha+(n-1)\beta]$

$$= \frac{\sin\frac{n\beta}{2}}{\sin\frac{\beta}{2}} \sin\left\{\alpha + \left(\frac{n-1}{2}\right)\beta\right\}$$

$\cos\alpha + \cos(\alpha+\beta) + \cos(\alpha+2\beta) + \cdots + \cos[\alpha+(n-1)\beta]$

$$= \frac{\sin\frac{n\beta}{2}}{\sin\frac{\beta}{2}} \cos\left[\alpha + \left(\frac{n-1}{2}\right)\beta\right]$$

Then answer the following questions.

44. $\sum_{r=1}^{n-1} \cos^2\left(\frac{r\pi}{n}\right)$ is equal to

(A) $\frac{n}{2}$ (B) $\frac{n}{2} - 1$ (C) $\frac{n}{2} + 1$ (D) $\frac{3n}{2}$

Soln $\sum_{r=1}^{n-1} \cos^2\left(\frac{r\pi}{n}\right) = \sum_{r=1}^{n-1}\left(\frac{1 + \cos\frac{2r\pi}{n}}{2}\right) = \frac{1}{2}\left(\sum_{r=1}^{n-1} 1 + \sum_{r=1}^{n-1} \cos\frac{2r\pi}{n}\right)$

$$= \frac{1}{2}\left[(n-1) + \cos\frac{2\pi}{n} + \cos\frac{4\pi}{n} + \cdots + \cos\frac{2(n-1)\pi}{n}\right]$$

$$= \frac{1}{2}\left[(n-1) + \frac{\sin (n-1)\frac{\pi}{n}}{\sin \frac{\pi}{n}} \cos\left[\frac{2\pi}{n} + (n-2)\frac{\pi}{n}\right]\right]$$

$$= \frac{1}{2}\left[n-1 + \frac{\sin\left(\pi - \frac{\pi}{n}\right)}{\sin \frac{\pi}{n}} \cos\left[\cancel{\frac{2\pi}{n}} + \pi - \cancel{\frac{2\pi}{n}}\right]\right]$$

$$= \frac{1}{2}\left[n-1 + \frac{\cancel{\sin \frac{\pi}{n}}}{\cancel{\sin \frac{\pi}{n}}} \cos \pi\right] = \frac{1}{2}[n-1-1]$$

$$= \frac{n-2}{2} = \frac{n}{2} - 1 \quad \underline{\text{Ans.}}\ (B)$$

45. $\sum_{r=2}^{n} \sin r\alpha$, where $(n+2)\alpha = 2\pi$ is

(A) 0 (B) 1 (C) 2 (D) 3

$\underline{\text{Soln.}}$ $\sum_{r=2}^{n} \sin r\alpha = \sin 2\alpha + \sin 3\alpha + \cdots\cdots + \sin n\alpha$

$$= \frac{\sin (n-1)\frac{\alpha}{2}}{\sin \frac{\alpha}{2}} \sin\left[2\alpha + (n-2)\frac{\alpha}{2}\right]$$

$[\because$ total no. of terms $= n-1]$

and $(n+2)\alpha = 2\pi \Rightarrow n\alpha = 2\pi - 2\alpha$

$\therefore \frac{n\alpha}{2} = \pi - \alpha$

$$= \frac{\sin\left(\frac{n\alpha}{2} - \frac{\alpha}{2}\right)}{\sin \frac{\alpha}{2}} \sin\left[2\alpha + \frac{n\alpha}{2} - \alpha\right]$$

$$= \frac{\sin\left(\pi - \alpha - \frac{\alpha}{2}\right)}{\sin \frac{\alpha}{2}} \sin \pi = 0 \quad \underline{\text{Ans.}}\ (A)$$

46. $\sqrt{1+\cos\alpha} + \sqrt{1+\cos 2\alpha} + \sqrt{1+\cos 3\alpha} + \cdots + \sqrt{1+\cos n\alpha}$ is equal to; where $\alpha \in \left(-\frac{\pi}{n}, \frac{\pi}{n}\right)$

(A) $\sqrt{2}\, \frac{\sin \frac{n\alpha}{4}}{\sin \alpha/4} \cos\left\{(n+1)\frac{\alpha}{4}\right\}$ (B) $\sqrt{2}\, \frac{\cos \frac{n\alpha}{4}}{\sin \frac{\alpha}{4}} \cos\left\{(n+1)\frac{\alpha}{4}\right\}$

(C) $\sqrt{2}\,\dfrac{\sin\frac{n\alpha}{4}}{\frac{\alpha}{4}}\sin\left\{(n+1)\frac{\alpha}{4}\right\}$ (D) $\sqrt{2}\,\dfrac{\cos\frac{n\alpha}{4}}{\cos\frac{\alpha}{4}}\cos\left[(n+1)\frac{\alpha}{4}\right]$

Soln: $\because\ 1+\cos 2\theta = 2\cos^2\theta \quad \therefore \sqrt{1+\cos 2\theta} = \sqrt{2}\cos\theta$

$\therefore$ Above expression reduces to

$\sqrt{2}\left[\cos\frac{\alpha}{2} + \cos\alpha + \cos\frac{3\alpha}{2} + \text{------} + \cos\frac{n\alpha}{2}\right]$

$= \sqrt{2}\,\dfrac{\sin\frac{n\alpha}{4}}{\sin\frac{\alpha}{4}}\cos\left[\frac{\alpha}{2}+(n-1)\frac{\alpha}{4}\right] = \sqrt{2}\,\dfrac{\sin\frac{n\alpha}{4}}{\sin\frac{\alpha}{4}}\cos\left\{(n+1)\frac{\alpha}{4}\right\}$

$\therefore$ Ans. (A)

SECTION - IV

Integer Answer Type

This section contains 10 questions. The answer to each of the questions is a single digit integer, ranging from 0 to 9. The appropriate bubbles below the respective question numbers in the ORS have to be darkened. For example, if the correct answers to question numbers X, Y and Z (say) are 6, 0 and 9, respectively, then the correct darkening of bubbles.

47. The value of $\tan 225^\circ \cot 405^\circ + \tan 765^\circ \cot 675^\circ$ is __

Soln: $\tan(180+45^\circ)\cot(360+45^\circ) + \tan(720+45^\circ)\cot(720^\circ-45^\circ)$

$= \tan 45^\circ \cot 45^\circ + \tan 45^\circ(-\cot 45^\circ) = 1\times 1 + 1\times(-1)$

$= 0$

48. The value of expression

$\dfrac{(\cos\theta - \cos 3\theta)(\sin 8\theta + \sin 2\theta)}{(\sin 5\theta - \sin\theta)(\cos 4\theta - \cos 6\theta)}$ is __

Soln: $\dfrac{2\sin 2\theta \sin\theta \; 2\sin 5\theta \cos 3\theta}{2\cos 3\theta \sin 2\theta \; 2\sin 5\theta \sin\theta} = 1$ Ans.

49. The value of $16\cos\frac{2\pi}{15}\cos\frac{4\pi}{15}\cos\frac{8\pi}{15}\cos\frac{14\pi}{15}$ is –

Soln. $16\cos\left(\pi-\frac{\pi}{15}\right)\cos\frac{2\pi}{15}\cos\frac{4\pi}{15}\cos\frac{8\pi}{15}$

$$=-16\cos\frac{\pi}{15}\cos\frac{2\pi}{15}\cos\frac{4\pi}{15}\cos\frac{8\pi}{15}$$

$$=-16\times\frac{1}{2^4}\frac{\sin\frac{16\pi}{15}}{\sin\frac{\pi}{15}}=\frac{-\sin\left(\pi+\frac{\pi}{15}\right)}{\sin\frac{\pi}{15}}=\frac{\sin\frac{\pi}{15}}{\sin\frac{\pi}{15}}=1$$

$$\boxed{\because \cos\theta\cdot\cos2\theta\cdot\cos4\theta\cdots\cos2^{n-1}\theta=\frac{1}{2^n}\frac{\sin2^n\theta}{\sin\theta}}$$

50. The value of $16\cos\frac{2\pi}{15}\cos\frac{4\pi}{15}\cos\frac{8\pi}{15}\cos\frac{16\pi}{15}$ is

Soln. $16\cos\frac{2\pi}{15}\cos\frac{4\pi}{15}\cos\frac{8\pi}{15}\cos\frac{16\pi}{15}$

$$=16\times\frac{1}{2^4}\frac{\sin\frac{32\pi}{15}}{\sin\frac{2\pi}{15}}=\frac{\sin\left(2\pi+\frac{2\pi}{15}\right)}{\sin\frac{2\pi}{15}}=\frac{\sin\frac{2\pi}{15}}{\sin\frac{2\pi}{15}}$$

$$=1 \text{ Ans.}$$

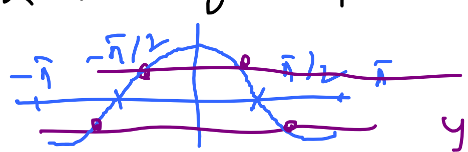
51. The number of ordered pair (α,β), where $\alpha,\beta\in(-\pi,\pi)$ satisfying $\cos(\alpha-\beta)=1$ and $\cos(\alpha+\beta)=\frac{1}{e}$ is_

Soln. $\cos(\alpha-\beta)=1 \Rightarrow \alpha-\beta=0 \Rightarrow \alpha=\beta.$

$$\cos(\alpha+\beta)=\frac{1}{e} \Rightarrow \cos2\alpha=\frac{1}{e}$$

$$\Rightarrow 2\cos^2\alpha-1=\frac{1}{e} \Rightarrow \cos^2\alpha=\frac{1+e}{2e}.$$

$\Rightarrow \cos\alpha=\pm\sqrt{\frac{1+e}{2e}}$ which will give 4 values in $[-\pi,\pi]$ as



$y=\sqrt{\frac{1+e}{2e}}$.

$y=-\sqrt{\frac{1+e}{2e}}$

Hence, there will be 4 pairs of (α,β).

52. The value of expression $\sqrt{3}\operatorname{cosec} 20^\circ - \sec 20^\circ$ is equal to

Soln. $\sqrt{3} \times \frac{1}{\sin 20} - \frac{1}{\cos 20} = 2\left[\frac{\sqrt{3}}{2}\frac{1}{\sin 20} - \frac{1}{2}\frac{1}{\cos 20}\right]$

$= 2\left[\frac{\sin 60^\circ}{\sin 20} - \frac{\cos 60}{\cos 20}\right]$

$= 2\left[\frac{\sin 60 \cos 20 - \cos 60 \sin 20}{\sin 20 \cos 20}\right] = \frac{2\times 2 \sin(60-20)}{2\sin 20 \cos 20}$

$= \frac{4 \sin 40}{\sin 40} = 4$ Ans.

53. Given that $\sqrt{3}\cos x - \sin x = \cot^{10} y + \tan^{10} y$. For the equality to hold $\tan y$ has got n number of values then n equal to ——

Soln. $2\left(\frac{\sqrt{3}}{2}\cos x - \frac{1}{2}\sin x\right) = \frac{1}{\tan^{10} y} + \tan^{10} y$

$2\left(\cos x \cos\frac{\pi}{6} - \sin x \sin\frac{\pi}{6}\right) = \frac{1}{\tan^{10} y} + \tan^{10} y$

$2\cos\left(x + \frac{\pi}{6}\right) = \frac{1}{\tan^{10} y} + \tan^{10} y$

Equality hold when LHS = RHS = 2

[$\because$ RHS ≥ 2 while LHS ≤ 2]

But RHS = 2 when $\tan^{10} y = 1 \Rightarrow \tan y = \pm 1$

$\therefore$ no. of values $\tan y$ can obtained $= 2$

54. The equation $\cos^{2010} x + \sin^{2012} x = 1$ has

$\frac{\pi}{2010} + \frac{\pi}{K}\left[n - \frac{1}{1005}\right]$, $n \in I$ as its general

Soln: Then K equals

Soln $\cos^{2010}x + \sin^{2012}x = 1$ is true for

$\cos x = 1$ or $\sin x = 1$

$\Rightarrow x = (2n+1)\frac{\pi}{2}$ or, $x = n\pi$

given Soln: $\frac{\pi}{2010} + \frac{\pi}{k}\left[n - \frac{1}{1005}\right] = \frac{\pi}{2010} + \frac{\pi}{k}\left[\frac{1005n-1}{1005}\right]$

$$= \frac{2\pi k + 2\pi[1005n-1]}{2010k}$$

$$= \frac{2\pi k + 2010\pi n - 2\pi}{2010k}$$

$$= \frac{(k + 1005n - 1)\pi}{1005k} = n\pi$$

when $k = 1$

55. The minimum value of $27^{\cos 2x} \cdot 81^{\sin 2x}$ is E. for some real x. Then $729(E)$ is equal to ——

Soln: Let $y = 27^{\cos 2x}\, 81^{\sin 2x}$

$$= 3^{3\cos 2x}\, 3^{4\sin 2x} = 3^{3\cos 2x + 4\sin 2x}$$

Min. value of $y = 3^{-5} = \frac{1}{243}$

i.e $E = \frac{1}{243}$ $\therefore 729E = 3$ Ans.

56. For the equation $(\cos p - 1)x^2 + (\cos p)x + \sin p = 0$ p has range (α, β) for some real x. Then total no. of integers in (α, β) including α and β if α and β are integers is ——

Soln. for real x, $D \geq 0$

$(\cos p)^2 - 4(\cos p - 1)\sin p \geq 0$

$\Rightarrow \cos^2 p + 4(1 - \cos p)\sin p \geq 0$

$\Rightarrow$ which will be true if $\sin p \geq 0$

$\Rightarrow p \in [0, \pi]$

Range of p is $[0, \pi]$

Then total no. of integers in (α, β) including α and β means the no. of integers in $[0, \pi] = 4$ $(0, 1, 2, 3)$

4 Ans.