Solution Of Self Evaluation Test-02 (Paper-II)

Topics :Transformation Formula & Trigonometric Equation)

SECTION - I

Straight Objective Type

This section contains 6 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

20. If $\sin\theta + \operatorname{cosec}\theta = 2$, then the value of $\sin^{2011}\theta + \operatorname{cosec}^{2011}\theta$ is

(A) 0 (B) 1 (C) -2 (D) 2

Soln. $\sin\theta + \operatorname{cosec}\theta = 2 \Rightarrow \sin\theta + \dfrac{1}{\sin\theta} = 2$

$\Rightarrow \sin^2\theta - 2\sin\theta + 1 = 0 \Rightarrow (\sin\theta - 1)^2 = 0 \Rightarrow \sin\theta = 1$

$\therefore \operatorname{cosec}\theta = 1$

$\therefore \sin^{2011}\theta + \operatorname{cosec}^{2011}\theta = (1)^{2011} + (1)^{2011} = 1 + 1 = 2$

21. If $a + b\tan\theta = \sec\theta$ and $b - a\tan\theta = 3\sec\theta$, then the value of $a^2 + b^2$ is

(A) 16 (B) 10 (C) 8 (D) 4

Soln. $a + b\tan\theta = \sec\theta \Rightarrow a\cos\theta + b\sin\theta = 1$ —— (i)

$b - a\tan\theta = 3\sec\theta \Rightarrow b\cos\theta - a\sin\theta = 3$ —— (ii)

Squaring and adding (i) and (ii) we get

$a^2 + b^2 = 10$

22. The number of solution of the equation $e^{\sin x} - e^{-\sin x} = 4$ is

(A) 1 (B) 2 (C) 4 (D) 0

Soln. $e^{\sin x} + \dfrac{1}{e^{\sin x}} = 4$ Put $e^{\sin x} = y$

$\therefore y + \dfrac{1}{y} = 4 \Rightarrow y^2 + 1 = 4y \Rightarrow y^2 - 4y + 1 = 0$

$\Rightarrow y = \dfrac{4 \pm \sqrt{16-4}}{2} = \dfrac{4 \pm \sqrt{12}}{2} = 2 \pm \sqrt{3}$

when $y = 2 + \sqrt{3} \Rightarrow e^{\sin x} = 2 + \sqrt{3}$ which is not possible as maximum value of $e^{\sin x} = e^1 = e$ and $e < 2+\sqrt{3}$

min value of $e^{\sin x} = e^{-1} = \frac{1}{e} \approx 0.34$

But $2-\sqrt{3} = 0.28 < \frac{1}{e}$ $\therefore$ $2-\sqrt{3}$ is also not possible. Hence, no soln. Ans. (D)

23. If $3\tan(\theta - 15^\circ) = \tan(\theta + 15^\circ)$, then θ is equals to

(A) $n\pi + \frac{\pi}{4}$, $n \in Z$ (B) $n\pi + \frac{\pi}{8}$, $n \in Z$

(C) $n\pi + \frac{\pi}{3}$, $n \in Z$ (D) $n\pi - \frac{\pi}{6}$, $n \in Z$

Soln.
$$\frac{\tan(\theta + 15^\circ)}{\tan(\theta - 15^\circ)} = \frac{3}{1} \Rightarrow \frac{\tan(\theta+15)+\tan(\theta-15^\circ)}{\tan(\theta+15)-\tan(\theta-15)} = \frac{3+1}{3-1}$$

$$\Rightarrow \frac{\frac{\sin(\theta+15)}{\cos(\theta+15)} + \frac{\sin(\theta-15)}{\cos(\theta-15)}}{\frac{\sin(\theta+15)}{\cos(\theta+15)} - \frac{\sin(\theta-15)}{\cos(\theta+15)}} = \frac{4}{2} \Rightarrow \frac{\sin[\theta+\cancel{15}+(\theta-\cancel{15})]}{\sin[\cancel{\theta}+15-\cancel{\theta}+15]} = 2$$

$$\Rightarrow \frac{\sin 2\theta}{\sin 30^\circ} = 2 \Rightarrow \sin 2\theta = 2\sin 30^\circ = 2\times\frac{1}{2} = 1$$

$$\Rightarrow \sin 2\theta = \sin\frac{\pi}{2} \Rightarrow 2\theta = 2n\pi + \frac{\pi}{2}$$

$$\therefore \theta = n\pi + \frac{\pi}{4}, n \in Z$$

$\therefore$ Ans. (A)

24. If $3\sin\theta + 4\cos\theta = 5$, then the value of $4\sin\theta - 3\cos\theta$ is

(A) 5 (B) 0 (C) 9 (D) 16

Soln. $3\sin\theta + 4\cos\theta = 5$

Squaring, $9\sin^2\theta + 16\cos^2\theta + 24\sin\theta\cos\theta = 25$

$\Rightarrow 9(1-\cos^2\theta) + 16(1-\sin^2\theta) + 24\sin\theta\cos\theta = 25$

$\Rightarrow 9\cos^2\theta + 16\sin^2\theta - 24\sin\theta\cos\theta = 0$

i.e. $(4\sin\theta - 3\cos\theta)^2 = 0 \Rightarrow 4\sin\theta - 3\cos\theta = 0$

25. If $\sec^2 5° + \sec^2 10° + \sec^2 15° + \sec^2 20° - \lambda = \tan^2 5° + \tan^2 10° + \tan^2 15° + \tan^2 20°$, then the value of λ is

(A) 1 (B) 2 (C) 3 (4) 4

Soln. $\lambda = \sec^2 5° - \tan^2 5° + \sec^2 10° - \tan^2 10° + \sec^2 15° - \tan^2 15° + \sec^2 20° - \tan^2 20°$

$\lambda = 1 + 1 + 1 + 1 = 4$ Ans $[\because \sec^2\theta - \tan^2\theta = 1]$

SECTION - IV

Integer Answer Type

This section contains 5 questions. The answer to each of the questions is a single digit integer, ranging from 0 to 9. The appropriate bubbles below the respective question numbers in the ORS have to be darkened. For example, if the correct answers to question numbers X, Y and Z (say) are 6, 0 and 9, respectively, then the correct darkening of bubbles.

26. If $\tan^2\theta = 1 + 2\tan^2\phi$, then the value of $2\cos^2\theta + \sin^2\phi + 1$ is ——

Soln. $\tan^2\theta = 1 + 2\tan^2\phi$

$\Rightarrow \sec^2\theta - 1 = 1 + 2(\sec^2\phi - 1)$

$\Rightarrow \sec^2\theta - 1 = 2\sec^2\phi - 1 \Rightarrow \sec^2\theta = 2\sec^2\phi$

$\Rightarrow \cos^2\phi = 2\cos^2\theta \Rightarrow 2\cos^2\theta - \cos^2\phi = 0$ Ans.

27. In $\triangle ABC$, if $\dfrac{\sin 2A + \sin 2B + \sin 2C}{\sin A + \sin B + \sin C} = K = \sin\dfrac{A}{2}\sin\dfrac{B}{2}\sin\dfrac{C}{2}$, then the value of K is ——

Soln. $$\frac{\sin 2A + \sin 2B + \sin 2C}{\sin A + \sin B + \sin C} = \frac{2\sin(A+B)\cos(A-B) + \sin 2C}{2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right) + \sin C}$$

$$= \frac{2\sin(\pi - C)\cos(A-B) + 2\sin C\cos C}{2\sin\left(\frac{\pi}{2} - \frac{C}{2}\right)\cos\left(\frac{A-B}{2}\right) + 2\sin\frac{C}{2}\cos\frac{C}{2}}$$

$$= \frac{\cancel{2}\sin C\left[\cos(A-B)+\cos\{\pi-(A+B)\}\right]}{\cancel{2}\cos\frac{C}{2}\left[\cos\frac{A-B}{2}+\sin\left(\frac{\pi}{2}-\frac{A+B}{2}\right)\right]}$$

$$= \frac{\sin C\left[\cos(A-B)-\cos(A+B)\right]}{\cos\frac{C}{2}\left[\cos\frac{A-B}{2}+\cos\frac{A+B}{2}\right]}$$

$$= \frac{\sin C\ \cancel{2}\sin A\sin B}{\cos\frac{C}{2}\ \cancel{2}\cos\frac{A}{2}\cos\frac{B}{2}}$$

$$= \frac{2\sin A/2\cos A/2\ 2\sin B/2\cos B/2\ 2\sin C/2\cos C/2}{\cos A/2\cos B/2\cos C/2}$$

$$= 8\sin A/2\sin B/2\sin C/2 \qquad \therefore K = 8$$

28. If $\sin^2 x + \cos^2 y = 2\cos^2 z \ \forall\ x, y, z \in [0, 2\pi]$ Such that sum of all possible values x, y, z is $k\pi$, then the value of k is ______

Sol^n max. value of $\sin^2 x + \cos^2 y$ is 2, the equation will be valid if $\sin^2 x = 1 \Rightarrow x = \frac{\pi}{2}, \frac{3\pi}{2}$

$\cos^2 y = 1 \Rightarrow y = 0, \pi, 2\pi,$

and $\cos^2 z = 1 \Rightarrow z = 0, \pi, 2\pi$

$\therefore$ Sum of all values of $x, y, z = K\pi$

$\therefore \frac{\pi}{2} + \frac{3\pi}{2} + 0 + \pi + 2\pi + 0 + \pi + 2\pi = K\pi$

$\Rightarrow 8\pi = k\pi \Rightarrow k = 8$ Ans

29. If $\sin x + \sin y = \sqrt{3}(\cos y - \cos x)$, then $\sin 3x + \sin 3y$ is ______

Sol^n $\cancel{2}\sin\frac{x+y}{2}\cos\frac{x-y}{2} = \sqrt{3}\times\cancel{2}\sin\frac{x+y}{2}\sin\frac{x-y}{2}$

$\sin\left(\frac{x+y}{2}\right)\left[\cos\left(\frac{x-y}{2}\right) - \sqrt{3}\sin\left(\frac{x-y}{2}\right)\right] = 0$

$$\sin\left(\frac{x+y}{2}\right)=0 \quad \text{or} \quad \cos\left(\frac{x-y}{2}\right)=\sqrt{3}\sin\left(\frac{x-y}{2}\right)$$

$$\tan\left(\frac{x-y}{2}\right)=\frac{1}{\sqrt{3}} \Rightarrow \frac{x-y}{2}=30^\circ$$

Now, $\sin 3x+\sin 3y$

$$=2\sin 3\left(\frac{x+y}{2}\right)\cos 3\left(\frac{x-y}{2}\right)$$

$$=2\sin 3\times 0 \cos 3\times 30^\circ = 2\sin 0\cos 90 = 0 \text{ Ans}$$

30. If $\frac{\cos(\alpha-\beta)}{\cos(\alpha+\beta)}+\frac{\cos(\gamma+\delta)}{\cos(\gamma-\delta)}=0$, then absolute value of $\tan\alpha\tan\beta\tan\gamma\tan\delta$ is ____

Soln. $$\frac{\cos(\alpha-\beta)}{\cos(\alpha+\beta)}=-\frac{\cos(\gamma+\delta)}{\cos(\gamma-\delta)}$$

using componendo and dividendo

$$\frac{\cos(\alpha+\beta)+\cos(\alpha-\beta)}{\cos(\alpha+\beta)-\cos(\alpha-\beta)}=\left[\frac{-\cos(\gamma-\delta)+\cos(\gamma+\delta)}{-\cos(\gamma-\delta)-\cos(\gamma+\delta)}\right]$$

$$\frac{\cancel{2}\cos\alpha\cos\beta}{\cancel{2}\sin\alpha\sin\beta}=\frac{\cancel{2}\sin\gamma\sin\delta}{\cancel{2}\cos\gamma\cos\delta}$$

$$\cot\alpha\cot\beta=\tan\gamma\tan\delta \Rightarrow \tan\gamma\tan\delta\tan\alpha\tan\beta=1$$

$$\Rightarrow \tan\alpha\tan\beta\tan\gamma\tan\delta=1 \text{ Ans.}$$

SECTION-III

Linked Comprehension Type

This section contains 2 paragraph. Based upon the first paragraph two multiple choice questions have to be answered and based upon the first paragraph three multiple choice questions have to be answered. . Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONLY ONE** is correct.

Paragraph for Question Nos. 31 to 33

Let $A+B=\frac{\pi}{4}$, such that $(1+\tan A)(1+\tan B)=2$

then answer the following questions.

31. If $(1+\tan\alpha)(1+\tan 4\alpha)=2$, $\alpha\in\left(0,\frac{\pi}{16}\right)$ then α is

(A) $\frac{\pi}{20}$ (B) $\frac{\pi}{30}$ (C) $\frac{\pi}{40}$ (D) $\frac{\pi}{80}$

Sol^n from given relation $\alpha+4\alpha=\frac{\pi}{4} \Rightarrow 5\alpha=\frac{\pi}{4}$

$\therefore \alpha=\frac{\pi}{20}$ Ans (A)

32. The value of $(1+\tan 1^\circ)(1+\tan 2^\circ)(1+\tan 3^\circ)(1+\tan 44^\circ)(1+\tan 43^\circ)(1+\tan 42^\circ)$ is

(A) 8 (B) 4 (C) 2 (D) 6

Sol^n $[(1+\tan 1^\circ)(1+\tan 44^\circ)][(1+\tan 2^\circ)(1+\tan 43^\circ)][(1+\tan 3^\circ)(1+\tan 42^\circ)]$

$=2\times 2\times 2=8$ Ans. (A)

33. The value of $(1+\tan 382^\circ)(1+\tan 203^\circ)$ is

(A) $\frac{1}{2}$ (B) 2 (C) 4 (D) $\frac{1}{4}$

Sol^n. $\tan(585^\circ)=\tan(382^\circ+203^\circ)$

$$\tan(540^\circ+45^\circ)=\frac{\tan 382^\circ+\tan 203^\circ}{1-\tan 382^\circ\tan 203^\circ}$$

$$1=\frac{\tan 382^\circ+\tan 203^\circ}{1-\tan 382^\circ\tan 203^\circ}$$

$\therefore \tan 382^\circ+\tan 203^\circ+\tan 382^\circ\tan 203^\circ=1$

$1+\tan 382^\circ+\tan 203^\circ(1+\tan 382^\circ)=1+1$

$(1+\tan 382^\circ)(1+\tan 203^\circ)=2$ Ans (B)

Paragraph for question No.s 34 to 36.

Sometimes eliminating θ between two equations involving x and y. We shall get interesting results in co-ordinate geometry. For example eliminating θ between $x\cos\theta - y\sin\theta = a$ and $x\sin\theta + y\cos\theta = b$ we get $x^2+y^2 = a^2+b^2$ which shows that the point (x, y) lies on the circle in x-y plane. keeping in mind, the equations $x^2+y^2=a^2$, $y^2=4ax$, $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ represent circle, parabola, ellipse and hyperbola respectively.

Answer the following questions

34. If $x\sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta$ and $x\sin\theta - y\cos\theta = 0$, then (x, y) lies on

(A) Circle (B) A parabola (C) An ellipse

(D) A hyperbola.

Soln. $x\sin\theta - y\cos\theta = 0 \Rightarrow x\sin\theta = y\cos\theta$

$\Rightarrow \sin\theta = \frac{y}{x}\cos\theta$ — (1)

$\therefore x\sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta$

$\Rightarrow \cancel{x}\left(\frac{y^3\cos^3\theta}{x^{\cancel{3}\,2}}\right) + y\cos^3\theta = \frac{y}{x}\cos\theta\cos\theta.$

$\Rightarrow \cancel{y}\cos^3\theta\,(y^2+x^2) = x\cancel{y}\cos^2\theta$

$\Rightarrow \cos\theta = \frac{x}{x^2+y^2}$ — (i)

$\sin\theta = \frac{y}{x}\times\frac{x}{x^2+y^2}$

$= \frac{y}{x^2+y^2}$ — (ii)

$\Rightarrow$ Squaring and adding

$\frac{x^2}{(x^2+y^2)^2} + \frac{y^2}{(x^2+y^2)^2} = 1 \Rightarrow x^2+y^2 = 1$

which is circle Ans. (A)

35. If $\frac{x}{a\cos\theta} = \frac{y}{b\sin\theta}$ and $\frac{ax}{\cos\theta} - \frac{by}{\sin\theta} = a^2-b^2$

Then (x, y) lies on

(A) Circle (B) A Parabola (C) An ellipse

(D) A hyperbola.

Soln $\frac{x}{a\cos\theta} = \frac{y}{b\sin\theta}$ $\Rightarrow \frac{ax}{\cos\theta} = \frac{a^2 y}{b\sin\theta}$ — (i)

$\therefore \frac{ax}{\cos\theta} - \frac{by}{\sin\theta} = a^2-b^2 \Rightarrow \frac{a^2y}{b\sin\theta} - \frac{by}{\sin\theta} = a^2-b^2$

$\Rightarrow \frac{(a^2-b^2)y}{b\sin\theta} = (a^2-b^2) \Rightarrow b\sin\theta = y$

$\Rightarrow \sin\theta = \frac{y}{b}$ — (ii) from (i) $\frac{x}{a\cos\theta} = 1$

$\Rightarrow \cos\theta = \frac{x}{a}$ — (iii) Squaring and adding

$\therefore \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ which is ellipse Ans (C)

36. If $\tan\theta + \sin\theta = p$ and $\tan\theta - \sin\theta = q$, then $(p^2-q^2)^2$ is

(A) $4\sqrt{pq}$ (B) $4pq$ (C) $16\sqrt{pq}$ (D) $16pq$

Soln. $\tan\theta + \sin\theta = p$ — (i)

$\tan\theta - \sin\theta = q$ — (ii)

$\therefore p^2 - q^2 = (\tan\theta + \sin\theta)^2 - (\tan\theta - \sin\theta)^2$

$= 4\tan\theta\sin\theta = 4\sqrt{pq}$ using (ii)

$\therefore pq = \tan^2\theta - \sin^2\theta = \tan^2\theta\sin^2\theta$ — (ii)

$\therefore (p^2-q^2)^2 = 16pq$ $\therefore$ Ans (D)

SECTION-IV

Matrix-Match Type

This section contains 2 questions. Each question contains statements given in two columns which have to be matched. The statements in **Column I** are labelled A, B, C and D, while the statements in **Column II** are labelled p, q, r, s and t. Any given statement in **Column I** can have correct matching with **One OR More** statement(s) in **Column II**. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following example. If the correct matches are A-p, s and t; B-q and r; C-p and q; and D-s and t;

37. Match the following

Column I	Column II
(A) If $A+B=\frac{5\pi}{4}$, then the value of $(1+\tan A)(1+\tan B)$ is	(p) 4
(B) The value of $\cos^4\frac{\pi}{8}+\cos^4\frac{3\pi}{8}+\cos^4\frac{5\pi}{8}+\cos^4\frac{7\pi}{8}+\frac{1}{2}$	(q) 6
(C) The value of $\tan 75^\circ+\cot 75^\circ$ is	(r) $\frac{3}{2}$
(D) The value of $\sqrt{3}\operatorname{cosec}20^\circ-\sec 20^\circ$ is	(s) $\frac{2}{\sqrt{3}}$
	(t) 2

Soln
(A) $(A+B)=\frac{5\pi}{4} \Rightarrow \tan(A+B)=\tan\frac{5\pi}{4}$

t

$\Rightarrow \frac{\tan A+\tan B}{1-\tan A\tan B}=1 \Rightarrow (1+\tan A)(1+\tan B)=2$

Short cut :- put $A=\pi$, $B=\frac{\pi}{4} \Rightarrow (1+0)(1+1)=2$

(B) $\cos^4\frac{\pi}{8}+\cos^4\frac{3\pi}{8}+\cos^4\frac{5\pi}{8}+\cos^4\frac{7\pi}{8}+\frac{1}{2}$

t

$=\cos^4\frac{\pi}{8}+\cos^4\left(\frac{\pi}{2}-\frac{\pi}{8}\right)+\cos^4\left(\frac{\pi}{2}+\frac{\pi}{8}\right)+\cos^4\left(\pi-\frac{\pi}{8}\right)+\frac{1}{2}$

$=\cos^4\frac{\pi}{8}+\sin^4\frac{\pi}{8}+\sin^4\frac{\pi}{8}+\cos^4\frac{\pi}{8}+\frac{1}{2}$

$=2\left[\cos^4\frac{\pi}{8}+\sin^4\frac{\pi}{8}\right]+\frac{1}{2}=2\left[\left(\cos^2\frac{\pi}{8}\right)^2+\left(\sin^2\frac{\pi}{8}\right)^2\right]+\frac{1}{2}$

$=2\left[\left(\cos^2\frac{\pi}{8}+\sin^2\frac{\pi}{8}\right)^2-2\sin^2\frac{\pi}{8}\cos^2\frac{\pi}{8}\right]+\frac{1}{2}$

$= 2 \times 1 - \left(2 \sin\frac{\pi}{8} \cos\frac{\pi}{8}\right)^2 + \frac{1}{2}$

$= 2 - \left(\sin\frac{\pi}{4}\right)^2 + \frac{1}{2} = 2 - \left(\frac{1}{\sqrt{2}}\right)^2 + \frac{1}{2}$

$= 2 - \cancel{\frac{1}{2}} + \cancel{\frac{1}{2}} = 2$ Ans.

(C) $\tan 75^\circ + \cot 75^\circ = \frac{\sqrt{3}+1}{\sqrt{3}-1} + \frac{\sqrt{3}-1}{\sqrt{3}+1}$

p

$= \frac{(\sqrt{3}+1)^2 + (\sqrt{3}-1)^2}{3-1} = \frac{8}{2} = 4$

(D) $\sqrt{3}\,\mathrm{cosec}\, 20^\circ - \sec 20^\circ$

p

$= 2\left[\frac{\frac{\sqrt{3}}{2}}{\sin 20} - \frac{\frac{1}{2}}{\cos 20}\right] = 2\left[\frac{\sin 60}{\sin 20} - \frac{\cos 60}{\cos 20}\right]$

$= 2\left[\frac{\sin(60-20)}{\sin 20 \cos 20}\right] = 2\,\frac{\sin 40}{\sin 20 \cos 20} = \frac{2 \times 2 \sin 20 \cos 20}{\sin 20 \cos 20}$

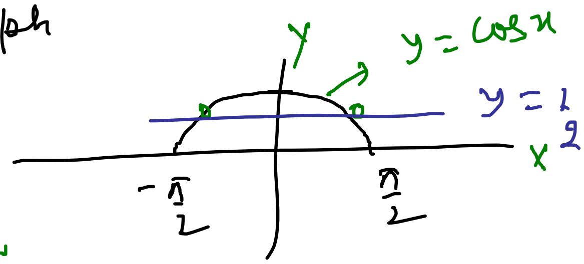
$= 4$ Ans.

38. Match the following

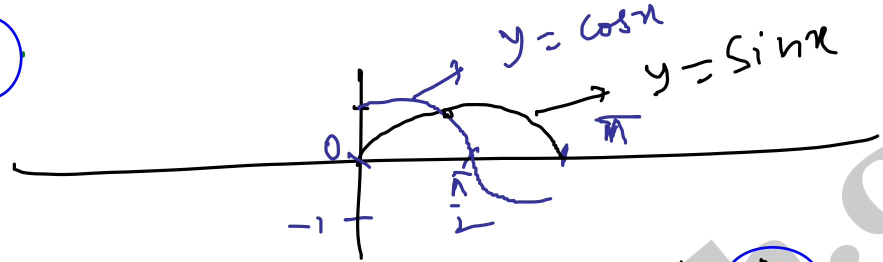
Column I	Column II
(A) The number of points of intersection of $2y = 1$ and $y = \cos x$ in $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	(p) 5
(B) The number of points of intersection of $y = \cos x$ and $y = \sin x$ in $x \in [0, \pi]$ is	(q) 3
(C) The number of points of intersection $y = \cot x$ and $y = \tan x$ in $[0, \pi]$ is	(r) 0
(D) The number of points of intersection $y = \sec x$ and $y = \mathrm{cosec}\, x$ in $[0, 2\pi]$ is	(s) 2
	(t) 1

Soln (A) By graph

no. of points of intersection is 2

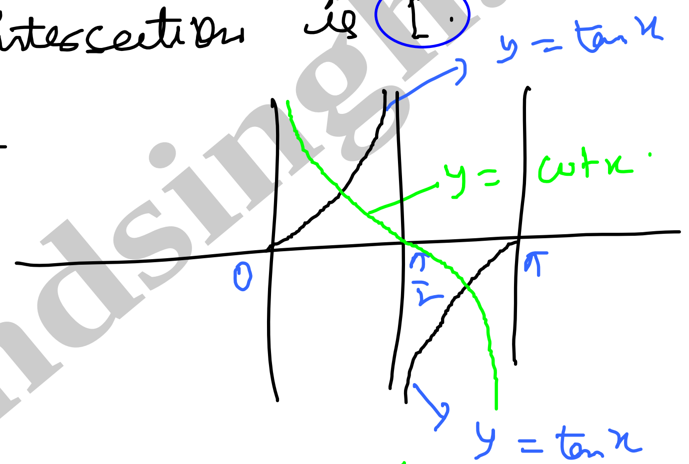


(B)

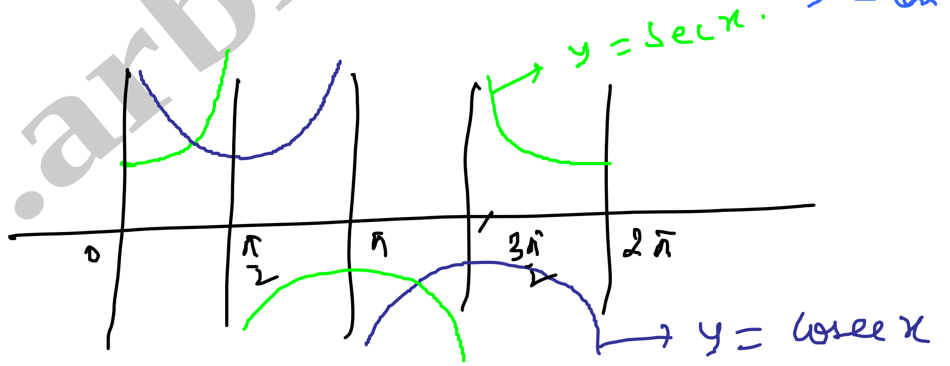


No. of points of intersection is 1.

(C) No. of point of intersection are 2



(D)



No. of points of intersection = 2