Solution Of Self Evaluation Test-01 (Paper-I)

Topics :Limit, Continuity , Differentiability and Differentiation

SECTION - I

Straight Objective Type

This section contains 8 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

29. $\lim_{x\to 0} \dfrac{\tan x - \sin x}{x^3}$ is

(a) 0 (b) 1/2 (c) 2 (d) None of these

Soln. $\lim_{x\to 0} \dfrac{\tan x(1-\cos x)}{x^3} = \lim_{x\to 0} \dfrac{\tan x}{x} \dfrac{2\sin^2\frac{x}{2}}{x^2}$

$\lim_{x\to 0} \dfrac{\tan x}{x} \times \cancel{2}\left(\dfrac{\sin\frac{x}{2}}{\frac{x}{2}}\right)^2 \times \dfrac{1}{\cancel{4}\,2} = 1\times(1)^2\times\frac{1}{2} = \frac{1}{2}$

Method 2. using expansion;

$\because \tan x = x + \dfrac{x^3}{3} + \dfrac{2}{15}x^5 + \ldots$

$\sin x = x - \dfrac{x^3}{3!} + \dfrac{x^5}{5!} - \ldots$

$\lim_{x\to 0} \dfrac{\tan x - \sin x}{x^3} = \lim_{x\to 0} \dfrac{\left(\frac{1}{3}+\frac{1}{3!}\right)x^3 + \left(\frac{2}{15}-\frac{1}{5!}\right)x^5 + \ldots}{x^3}$

$\lim_{x\to 0} \left(\frac{1}{3}+\frac{1}{6}\right) + \left(\frac{2}{15}-\frac{1}{5!}\right)x^2 + \ldots$

$= \frac{1}{3}+\frac{1}{6} = \frac{1}{2}$ Ans.

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30) The integer n for which $\lim_{x \to 0} \frac{(\cos x - 1)(\cos x - e^x)}{x^n}$ is a finite non-zero number is

(a) 1 (b) 2 (c) 3 (d) 4

Soln. using expansion

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - - - -$$

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + - - -$$

$$\lim_{x \to 0} \frac{(\cos x - 1)(\cos x - e^x)}{x^n}$$

$$\lim_{x \to 0} \frac{\left(-\frac{x^2}{2!} + \frac{x^4}{4!} - - -\right)\left[\left(\cancel{1} - \frac{x^2}{2!} + \frac{x^4}{4!} - - -\right) - \left(\cancel{1} + x + \frac{x^2}{2!} - -\right)\right]}{x^n}$$

$$\lim_{x \to 0} \frac{x^2\left(-\frac{1}{2!} + \frac{x^2}{4!} - - -\right)\left(-x - x^2 - \frac{x^3}{3!} + \frac{x^4}{4!} - -\right)}{x^n}$$

$$\lim_{x \to 0} \frac{x^3\left(-\frac{1}{2!} + \frac{x^2}{4!} - - -\right)\left(1 - x + - - -\right)}{x^n}$$

$$\lim_{x \to 0} \frac{\left(-\frac{1}{2!} + \frac{x^2}{4!} - - -\right)\left(1 - x + \cdot - -\right)}{x^{n-3}} \text{ is Finite}$$

and non zero, if $n - 3 = 0 \Rightarrow n = 3$

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31. $\lim_{x \to -1^+} \dfrac{\sqrt{\pi} - \sqrt{\cos^{-1} x}}{\sqrt{x+1}}$ is equal to

(a) $\dfrac{1}{\sqrt{2}}$ (b) $\dfrac{1}{\sqrt{2\pi}}$ (c) $\dfrac{1}{\sqrt{\pi}}$ (d) None

Soln. Since it is in form of $\left(\frac{0}{0}\right)$, therefore

using L'H Rule

$$\lim_{x \to -1^+} \frac{0 - \frac{1}{2\sqrt{\cos^{-1} x}} \times \frac{-1}{\sqrt{1-x^2}}}{\frac{1}{2\sqrt{1+x}}} = \lim_{x \to -1^+} \frac{1}{\sqrt{\cos^{-1} x}\,\sqrt{1-x}}$$

$$= \frac{1}{2\sqrt{\pi}} \text{ Ans.}$$

32. $\lim_{x \to 0} \dfrac{\sin(\pi \cos^2 x)}{x^2}$ equals

(A) $-\pi$ (B) π (C) $\dfrac{\pi}{2}$ (D) None

Soln. $\lim_{x \to 0} \dfrac{\sin(\pi \cos^2 x)}{x^2}$ $\left(\frac{0}{0} \text{ form}\right)$

Applying LH Rule, we have

$$\lim_{x \to 0} \frac{\cos(\pi \cos^2 x)\ \pi \sin 2x}{2x}$$

$$\lim_{x \to 0} \pi \cos(\pi \cos^2 x) \left(\frac{\sin 2x}{2x}\right) = \pi(+1) \times 1 = +\pi$$

33. If $g(x)$ is a Polynomial satisfying $g(x)\,g(y) =$ $g(x) + g(y) + g(xy) - 2$ for all real x and y and

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$g(2) = 5$, Then $\lim_{x \to 3} g(x)$ is

(A) 9 (B) 25 (C) 10 (D) None.

Soln. Put $x = y = 0$

Then $[g(0)]^2 = 3g(0) - 2$

$\Rightarrow (g(0))^2 - 3g(0) + 2 = 0$

$\Rightarrow (g(0) - 2)(g(0) - 1) = 0$

$\Rightarrow g(0) = 2$ or $g(0) = 1$ — (1)

when $x = 1$, $y = 0$

$g(1)\,g(0) = g(1) + g(0) + g(0) - 2.$

$g(1)[g(0) - 1] = 2[g(0) - 1]$

$[g(0) - 1][g(1) - 2] = 0$

$g(0) = 1$, $g(1) = 2$ — (1 1)

from (1) and (2) $g(0) = 1$, $g(1) = 2$

and $g(2) = 5$ given,

$\Rightarrow g(x) = x^2 + 1 \quad \therefore \lim_{x \to 3} g(x) = 3^2 + 1 = 10$

34. The Set of all Points, where the function $f(x) = \sqrt{1 - e^{-x^2}}$ is differentiable is

(A) $(0, \infty)$ (B) $(-\infty, \infty)$ (C) $(-\infty, \infty) - \{0\}$ (D) $(-1, \infty)$

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Sol^n. $f'(x) = \dfrac{1}{2\sqrt{1-e^{-x^2}}} \times e^{-x^2} \times (-2x)$

Clearly, $f'(x)$ does not exists when $x=0$

$\therefore$ $f(x)$ is differentiable in $(-\infty, \infty) - \{0\}$

35. $\lim\limits_{x \to \frac{\pi}{3}} \dfrac{\tan^3 x - 3\tan x}{\cos\left(x + \frac{\pi}{6}\right)}$ is equal to

(A) 4 (B) -6 (C) -24 (D) 24

Sol^n. $\because$ Using L-H Rule.

$$\lim_{x \to \frac{\pi}{3}} \frac{3\tan^2 x \sec^2 x - 3\sec^2 x}{-\sin\left(x + \frac{\pi}{6}\right)}$$

$$= \frac{3 \times (\sqrt{3})^2 \times (2)^2 - 3 \times (2)^2}{-1}$$

$$= \frac{3 \times 3 \times 4 - 3 \times 4}{-1}$$

$$= \frac{36 - 12}{-1} = -24$$

Ans.

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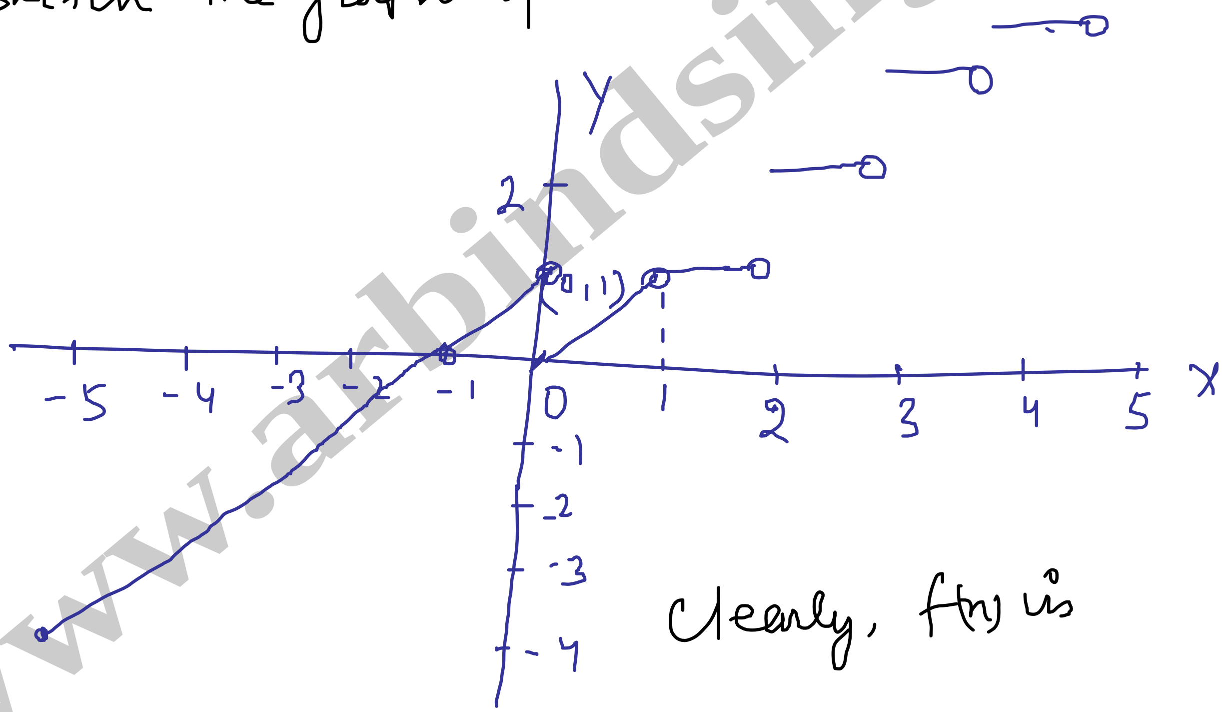
36. Let $f(x) = \begin{cases} x+1, & -5 < x < -1 \\ \{x\}, & -1 \le x < 1 \\ [x], & 1 \le x < 5 \end{cases}$ where $\{x\} = x - [x]$

and [ ] is greatest integer function. The number of points of discontinuity of $f(x)$ in $(-5, 5)$ is

(A) 3 (B) 6 (C) 4 (d) 5

Soln. Sketch the graph of above function in $(-5, 5)$



Clearly, $f(x)$ is discontinuous at 0, 2, 3, 4

Hence, no. of point of discontinuity is 4.

SECTION - II

Multiple Correct Answer Type

This section contains 4 multiple correct answer(s) type questions. Each question has 4 choices (A), (B), (C) and (D), out of which **ONE OR MORE** is/are correct.

37. If $f(x) = \left(\frac{x}{2+x}\right)^{2x}$, then

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(A) $\lim_{x\to\infty} f(x) = e^{-6}$ (B) $\lim_{x\to\infty} f(x) = 2$

(C) $\lim_{x\to\infty} f(x) = e^{-4}$ (D) $\lim_{x\to 1} f(x) = \frac{1}{9}$

Soln. $f(x) = \left(\frac{x}{2+x}\right)^{2x} = \left(\frac{1}{1+\frac{2}{x}}\right)^{2x}$

C, D.

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \left(\frac{1}{1+\frac{2}{x}}\right)^{2x} \quad (1^{\infty})$$

$$= e^{\lim_{x\to\infty} 2x\left(\frac{1}{1+\frac{2}{x}} - 1\right)} = e^{\lim_{x\to\infty}\left(\frac{-\frac{2}{x}}{1+\frac{2}{x}}\right)2x}$$

$$= e^{\lim_{x\to\infty} \frac{-4}{1+\frac{2}{x}}} = e^{-4}$$ $\therefore$ A is not correct and (C) is correct

$$\lim_{x\to 1} f(x) = \lim_{x\to 1} \left(\frac{x}{x+2}\right)^{2x} = \left(\frac{1}{3}\right)^{2} = \frac{1}{9}$$

$\therefore$ (D) is correct.

38. Let $f(x) = [x] + [-x]$, Then for any integer n and $k \in R - I$

(A) $\lim_{x\to n} f(x)$ exists (B) $\lim_{x\to k} f(x)$ exists

(C) f is continuous at $x = n$ (D) f is continuous at $x = k$

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Soln: We know, $[-x] = -1 - [x] \ \forall x \in R - I$

$\Rightarrow [x] + [-x] = -1 \ \forall x \in R - I$

and $[-x] = -[x] \ \forall x \in I$

$\Rightarrow [x] + [-x] = 0 \ \forall x \in I$

$\therefore f(x) = -1, \quad x \in R - I$

$\quad = 0, \quad x \in I$

$\because K \in R - I \ \therefore \lim_{x \to K} f(x) = -1$

for any integer n, $\lim_{x \to n} f(x) = 0$

and Clearly $f(x)$ is continuous at K as $K \in R - I$

$\therefore$ A, B, D are correct answer.

$\lim_{x \to n} f(x) = -1$ and $f(n) = 0$

$\therefore$ Hence, $\lim_{x \to n} f(x) \neq f(n)$

$\therefore f(x)$ is discontinuous at $x = n$.

39. If $\lim_{x \to 0} \dfrac{a e^{x} - b \cos x + c e^{-x}}{x \sin x} = 2$, then

(A) $a = e$ (B) $c = 1$ (C) $a = 2$ (D) $b = 2$

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Soln. We know:

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots$$

$$e^{-x} = 1 - \frac{x}{1!} + \frac{x^2}{2!} - \frac{x^3}{3!} + \cdots$$

$$\therefore \lim_{x\to 0} \frac{a\left(1+\frac{x}{1!}+\frac{x^2}{2!}+\cdots\right) - b\left[1-\frac{x^2}{2!}+\frac{x^4}{4!}-\cdots\right] + c\left(1-x+\frac{x^2}{2!}-\cdots\right)}{x\left[x-\frac{x^3}{3!}+\frac{x^5}{5!}-\cdots\right)} = 2$$

$$\lim_{x\to 0} \frac{(a-b+c) + (a-c)x + \left(\frac{a}{2!}+\frac{b}{2!}+\frac{c}{2!}\right)x^2 + \cdots}{x^2\left[1-\frac{x^2}{3!}+\frac{x^4}{5!}-\cdots\right]} = 2$$

This is possible if and only if

$a - b + c = 0$ — (i) $\Rightarrow b = a + c \Rightarrow b = 2a$

$a - c = 0$ — (ii) $\Rightarrow a = c$ $\quad (\because a = c)$

$\frac{a+b+c}{2!} = 2$ — (iii) $\Rightarrow a + b + c = 4$

$a + 2a + a = 4 \Rightarrow a = 1$

$b = 2$

$c = 1$

Hence, A, B and D are correct.

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40. If $y = \tan^{-1}\frac{1}{1+x+x^2} + \tan^{-1}\frac{1}{x^2+3x+3} + \tan^{-1}\frac{1}{x^2+5x+7}$ + ---- upto n terms, Then $y'(0)$ is equal to

(A) $-\frac{1}{1+n^2}$ (B) $-\frac{n^2}{1+n^2}$

(C) $\frac{n}{1+n^2}$ (D) None of these

Soln.

$$y = \tan^{-1}\frac{1}{1+x+x^2} + \tan^{-1}\frac{1}{x^2+3x+3} + \cdots$$

$$\therefore y = \tan^{-1}\frac{1}{1+x(1+x)} + \tan^{-1}\frac{1}{1+(x+2)(x+1)} + \cdots$$

$$= \tan^{-1}\frac{(x+1)-x}{1+x(1+x)} + \tan^{-1}\frac{(x+2)-(x+1)}{1+(x+2)(x+1)} + \cdots$$

$$= \left[\tan^{-1}\cancel{(x+1)} - \tan^{-1}x\right] + \left[\tan^{-1}\cancel{(x+2)} - \tan^{-1}\cancel{(x+1)}\right] + \cdots$$

$$\cdots + \left[\tan^{-1}(x+n) - \tan^{-1}\cancel{(x+n-1)}\right]$$

$$y = \tan^{-1}(x+n) - \tan^{-1}x.$$

$$\frac{dy}{dx} = \frac{1}{1+(x+n)^2} - \frac{1}{1+x^2}$$

$$\therefore \frac{dy}{dx} \text{ at } x=0 = \frac{1}{1+n^2} - \frac{1}{1} = \frac{-n^2}{1+n^2}$$

$$y(0) = \frac{-n^2}{1+n^2}$$ Ans (B)

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47. Let $h(x) = \min\{x, x^2\}$ for every real number x. Then

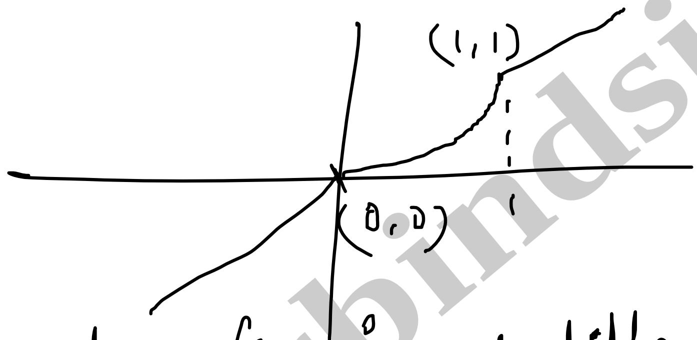
(A) h is continuous for all x

(B) h is differentiable for all x,

(C) $h'(x) = 1$, for all $x > 1$

(D) h is not differentiable at two value of x.

Sol^n. By graph, $h(x)$ will be as



Clearly, $f(x)$ is not differentiable at $x = 0, 1$ as there are two slopes on the curve can be determine.

$h(x)$ is continuous for all value of x

Also,
$$h(x) = x, \quad x \le 0$$
$$= x^2, \quad 0 \le x \le 1$$
$$= x \quad x \ge 1$$

$\therefore h'(x) = 1 \quad \forall\ x > 1$

Hence, A, C, D are correct.

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SECTION-III

Linked Comprehension Type

This section contains 2 paragraph. Based upon the first paragraph two multiple choice questions have to be answered and based upon the first paragraph three multiple choice questions have to be answered. . Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONLY ONE** is correct.

If L.H.D and R.H.D at $x=a$ exist and are equal, then function is differentiable at $x=a$, and hence, it is continuous at $x=a$ i.e

$$\lim_{h\to 0}\frac{f(a-h)-f(a)}{-h} = \lim_{h\to 0}\frac{f(a+h)-f(a)}{h} = f'(a)$$

$\Rightarrow f(x)$ is differentiable at $x=a$

42. $f(x) = \dfrac{x-[x]}{\{x\}+1}$ is

(A) Differentiable at $x=1$

(B) Discontinuous at $x=1$

(C) Continuous at $x=1$

(D) Non-differentiable at $x = \dfrac{2008}{2009}$

Soln. (B)

$$\lim_{h\to 0} f(1+h) = \lim_{h\to 0}\frac{1+h-[1+h]}{1+h-[1+h]+1}$$

$$= \lim_{h\to 0}\frac{\cancel{1}+h-\cancel{1}}{2+h-1} = \lim_{h\to 0}\frac{h}{1+h} = 0$$

$$\lim_{h\to 0} f(1-h) = \lim_{h\to 0}\frac{1-h-[1-h]}{1-h-[1-h]+1}$$

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$$= \lim_{h\to 0} \frac{1-h-0}{1-h-0+1} = \lim_{h\to 0} \frac{1-h}{2-h} = \frac{1}{2}$$

$\because f(1) = \frac{1-1}{0+1} = 0$

$\because f(1+h) \neq f(1-h)$

Hence $f(x)$ is discontinuous at $x=1$

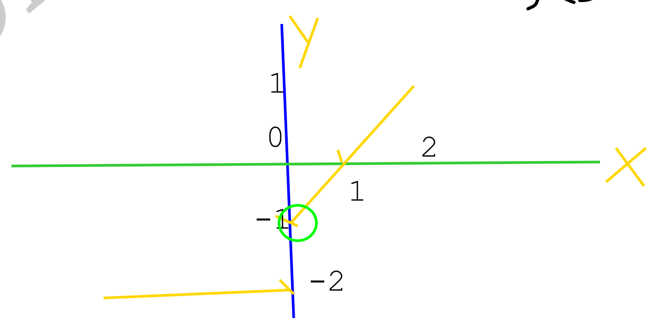
43. $f(x) = \begin{cases} -2, & -2 \le x \le 0 \\ x-1, & 0 < x \le 2 \end{cases}$ is

(A) Continuous at $x=0$ (B) Differentiable at $x=0$

(C) Discontinuous at $x=0$ (D) Non-differentiable at $x=1$

Soln. By graph, it is clear that $f(x)$ is

(C) discontinuous at $x=0$



Paragraph for Question Nos 44 to 46.

Let us consider the definite integral $I(x) = \int_{\phi(x)}^{\psi(x)} f(t)\,dt$

Newton - Leibnitz's formula states that

$$I'(x) = f(\psi(x))\cdot\psi'(x) - f(\phi(x))\cdot\phi'(x)$$

considering $l = \lim_{x\to 0} \dfrac{\int_0^x \frac{t^2\,dt}{(a+t^k)^{\frac{1}{k}}}}{bx - \sin x}$, where $k \in N$, $k \ge 2, a > 0$

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$h > 0$ and b is non zero quantity.

(44) If l exists and non-zero. Then b equals

(A) 7 (B) 4 (C) 2 (D) 1

Soln.
$$l = \lim_{x\to 0} \frac{\int_0^x \frac{t^2\,dt}{(a+t^h)^{\frac{1}{k}}}}{bx - \sin x}, \left(\frac{0}{0}\right)$$

using L-Hospital Rule,

$$= \lim_{x\to 0} \frac{\frac{x^2}{(a+x^h)^{\frac{1}{k}}}}{b - \cos x} = \lim_{x\to 0} \frac{x^2}{(a+x^h)^{\frac{1}{k}}(b-\cos x)} \quad -(1)$$

$$= \lim_{x\to 0} \frac{x^2}{(a+x^h)^{\frac{1}{k}}\left[b - \left(1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots\right)\right]}$$

$$l = \lim_{x\to 0} \frac{x^2}{(a+x^h)^{\frac{1}{k}}\left[(b-1) + \frac{x^2}{2!} - \frac{x^4}{4!} - \cdots\right]}$$

which will exists and non zero if $b-1=0$

$\Rightarrow b = 1$

* Detailed explanation for further refrence, when $b=1$

$$l = \lim_{x\to 0} \frac{\cancel{x^2}}{(a+x^h)^{\frac{1}{k}}\cancel{x^2}\left[\frac{1}{2!} - \frac{x^2}{4!} + \cdots\right]} = \frac{1}{a^{1/k}\left[\frac{1}{2!} - 0\right]}$$

$$= \frac{2}{a^{1/k}}$$

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45. If $k = 4$, $l = 3$ then the value of 'a' equals

(A) $\frac{16}{81}$ (B) $\frac{125}{27}$ (C) $\frac{81}{16}$ (D) $\frac{27}{125}$

Sol^n (A) from above detailed explanation

we have $l = \frac{2}{a^{\frac{1}{k}}} \Rightarrow a^{\frac{1}{k}} = \frac{2}{l} \Rightarrow a = \left(\frac{2}{l}\right)^k$

when $l = 3$, $k = 4 \Rightarrow a = \left(\frac{2}{3}\right)^4 = \frac{16}{81}$ Ans

46. If $k = 5$ and $a = 243$ and l exists, then the value of l is equal to

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{3}{2}$ (D) $\frac{2}{3}$

Sol^n (D) $\because l = \frac{2}{a^{\frac{1}{k}}} = \frac{2}{(243)^{\frac{1}{5}}} = \frac{2}{3}$ Ans

SECTION - IV

Integer Answer Type

This section contains 3 questions. The answer to each of the questions is a single digit integer, ranging from 0 to 9. The appropriate bubbles below the respective question numbers in the ORS have to be darkened. For example, if the correct answers to question numbers X, Y and Z (say) are 6, 0 and 9, respectively, then the correct darkening of bubbles.

47. A Square is inscribed in a circle of radius 1 unit, a circle is inscribed in this square then a square in this circle and the process is continued infinitely many times. The sum of areas

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of all squares is equal to ——

Soln 2.

$\because$ radius of circle = 1 unit

$\therefore$ diameter = 2 unit.

diagonal of square inscribed in circle

= diameter of circle = 2 unit

$\therefore$ Side of square = $\sqrt{2}$ unit and every time it will be multiply by $\frac{1}{\sqrt{2}}$

$\therefore$ Area = $(\sqrt{2})^2 + (1)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 + \cdots \infty$

$= 2 + 1 + \frac{1}{2} + \cdots$

$= \frac{2}{1-\frac{1}{2}} = 2 \times 2 = 4$

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48. Let $a_1 = 1$, $a_n = n(a_{n-1}+1)$ for $n = 2, 3, \cdots$

where $P_n = \left(1+\frac{1}{a_1}\right)\left(1+\frac{1}{a_2}\right)\cdots\left(1+\frac{1}{a_n}\right)$ then

$\lim_{n\to\infty} [P_n]$ equals, (where $[\cdot]$ denotes the greatest integer function)

Soln. 2

$\because a_n = n(a_{n-1}+1)$

$n=2$, $\therefore a_2 = 2(a_1+1) = 2(1+1) = 2\cdot 2$

$n=3$, $\therefore a_3 = 3(a_2+1) = 3\cdot(4+1) = 3\cdot 5$

$n=4$ $\therefore a_4 = 4(a_3+1) = 4(15+1) = 4\cdot 16$

$\therefore P_n = \left(1+\frac{1}{a_1}\right)\left(1+\frac{1}{a_2}\right)\left(1+\frac{1}{a_3}\right)\cdots\left(1+\frac{1}{a_n}\right)$

Put $n=2$ $\therefore P_2 = \left(1+\frac{1}{a_1}\right)\left(1+\frac{1}{a_2}\right)$

$= (1+1)\left(1+\frac{1}{4}\right) = \cancel{2}\times\frac{5}{\cancel{4}_2} = \frac{5}{2}$

$\therefore [P_2] = 2$ Ans. The same will proceeds for any value of n.

49. Let $l = \lim_{n\to\infty} \prod_{r=3}^{n} \left(\frac{r^3-8}{r^3+8}\right)$, Then $7l$ is equal to ——

Soln. 2. $l = \lim_{n\to\infty} \prod_{r=3}^{n} \frac{(r-2)(r^2+2r+4)}{(r+2)(r^2-2r+4)}$

$\therefore l = \lim_{n\to\infty} \prod_{r=3}^{n} \left(\frac{r-2}{r+2}\right)\left(\frac{r^2+2r+4}{r^2-2r+4}\right)$

$= \lim_{n\to\infty} \left(\frac{1}{\cancel{5}}\cdot\frac{2}{\cancel{6}}\cdot\frac{3}{7}\cdot\frac{4}{8}\cdot\frac{\cancel{5}}{9}\cdot\frac{\cancel{6}}{10}\cdots\right.$

$\left. - \frac{n-6}{\cancel{n-2}}\cdot\frac{n-5}{n-1}\cdot\frac{n-4}{n}\cdot\frac{\cancel{n-3}}{n+1}\cdot\frac{\cancel{n-2}}{n+2}\right)$

$\left(\frac{\cancel{19}}{7}\cdot\frac{\cancel{28}}{12}\cdot\frac{\cancel{39}}{\cancel{19}}\cdot\frac{\cancel{52}}{\cancel{28}}\cdots \frac{(n-2)^2+2(n-2)+4}{(n-2)^2-2(n-2)+4}\right.$

$\left. \cdots \frac{(n-1)^2+2(n-1)+4}{(n-1)^2\cancel{-2(n-1)+4}}\cdot\frac{n^2+2n+4}{n^2\cancel{-2n}+4}\right)$

$$= \lim_{n\to\infty} \frac{1\cdot 2\cdot \cancel{3}\cdot \cancel{4}\cdot (n^2+3)(n^2+2n+4)}{7\cdot \cancel{12}\,(n-1)\,n\,(n+1)(n+2)}$$

$$l = \lim_{n\to\infty} \frac{2}{7} \frac{\cancel{n^2}\left(1+\frac{3}{n^2}\right)\cancel{n^2}\left(1+\frac{2}{n}+\frac{4}{n^2}\right)}{\cancel{n^4}\left(1-\frac{1}{n}\right)\left(1+\frac{1}{n}\right)\left(1+\frac{2}{n}\right)}$$

$l = \frac{2}{7}$ $\therefore$ $7l = 2$ Ans.

50. If $y = 2x^2 \sin\left(\frac{1}{\{x-1\}}\right)$, then $\frac{dy}{dx}$ at $x=0$ is ___

Soln 0 $y = 2x^2 \sin\left(\frac{1}{x-1}\right)$

$$\frac{dy}{dx} = 4x \sin\left(\frac{1}{x-1}\right) + 2x^2 \cos\left(\frac{1}{x-1}\right)\left(\frac{-1}{(x-1)^2}\right)$$

$\frac{dy}{dx}$ at $x=0$ $= 0$

51. Let $f: R \to R$ be such that $f(1) = 3$ and $f'(1) = 6$, then logarithm of

$$\lim_{x\to 0}\left[\frac{f(1+x)}{f(1)}\right]^{\frac{1}{x}}$$ equals ____

Soln 2. $\lim_{x\to 0}\left[\frac{f(1+x)}{f(1)}\right]^{\frac{1}{x}}$ is in form of 1^∞

$$= e^{\lim_{x\to 0} \frac{1}{x}\left[\frac{f(1+x)}{f(1)} - 1\right]} = e^{\lim_{x\to 0} \frac{f(1+x) - f(1)}{x\cdot f(1)}}$$

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$$= e^{\lim_{x \to 0} \frac{f'(1)}{f(1)}} = e^{\frac{f'(1)}{f(1)}} = e^{\frac{6}{3}} = e^2$$

$$\therefore \text{logarithm of } \lim_{x \to 0} \left[\frac{f(1+x)}{f(1)}\right]^{\frac{1}{x}} = \log e^2 = 2$$

52. $f(x) = (x^2-1)\,|x^2-3x+2| + \cos|x|$ is not differentiable at ___

<u>Soln.</u> 2

$$f(x) = (x^2-1)\left|(x-1)(x-2)\right| + \cos|x|$$

$$= (x-1)^2(x+1)(x-2) + \cos x, \quad x<1$$

$$= -(x-1)^2(x+1)(x-2) + \cos x, \quad 1 \le x < 2$$

$$= (x-1)^2(x+1)(x-2) + \cos x, \quad 2 \le x$$

$$f(2) = \cos 2$$

$$f(2+h) = (1+h)^2(3+h)\,h + \cos(2+h)$$

$$f(2-h) = -(1-h)^2(3-h)(-h) + \cos(2-h)$$

$$= h(1-h)^2(3-h) + \cos(2-h)$$

$$\text{LHD } f'(2) = \lim_{h \to 0} \frac{f(2-h) - f(2)}{-h}$$

$$= \lim_{h \to 0} \frac{h(1-h)^2(3-h) + \cos(2-h) - \cos 2}{-h}$$

$$= \lim_{h \to 0} -(1-h)^2(3-h) - \frac{\cancel{2}\sin\left(2-\frac{h}{2}\right)\sin\frac{h}{2}}{\cancel{2} \times h/2}$$

$$= -3 - \sin 2$$

RHD $f'(2) = \lim_{h \to 0} \dfrac{f(2+h) - f(2)}{h} = 3 + \sin 2$

$\therefore$ $f(x)$ is not differentiable at $x = 2$

53. Let f and g be inverse of each other. If $f(1) = 5$ and $f'(1) = 4$, then $\dfrac{1}{g'(5)}$ is equal to ___

Soln 4. $\because$ f and g be inverse of each other

$\therefore$ $g(f(x)) = x \Rightarrow g'(f(x))\, f'(x) = 1$

when $x = 1$, $g'(f(1))\, f'(1) = 1$

$$g'(5) \times 4 = 1 \Rightarrow \frac{1}{g'(5)} = 4$$

54. $\lim_{x \to \infty} \left(\dfrac{x+6}{x+1}\right)^{\sqrt{x^2+1}} = e^{-k}$ then k is equal to ___

Soln 5. Put $x = -y$ when $x \to -\infty$, $y \to \infty$

$$\lim_{y \to \infty} \left(\frac{-y+6}{-y+1}\right)^{\sqrt{y^2+1}} = e^{-k}$$

$$= e^{\lim_{y \to \infty} \sqrt{y^2+1}\left(\frac{-y+6}{-y+1} - 1\right)} = e^{-k}$$

$$= \lim_{y \to \infty} y\sqrt{1 + \frac{1}{y^2}}\ \frac{-5}{\left(1 - \frac{1}{y}\right)} = -k \Rightarrow k = 5$$

55. Let $f(x) = x^3 + x^2 f'(1) + x f''(2) + f'''(3)$ $\forall x \in R$, then $f(1)$ is ___

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Soln. 4 $f(1) = 1 + f'(1) + f''(2) + f'''(3)$ —— (i)

$f'(x) = 3x^2 + 2x f'(1) + f''(2)$

when $x = 1 \Rightarrow f'(1) = 3 + 2f'(1) + f''(2)$

$f''(2) + f'(1) + 3 = 0$ —— (ii)

$f''(x) = 6x + 2f'(1)$

$f''(2) = 12 + 2f'(1)$ —— (iii)

$f'''(x) = 6$

$\therefore f'''(3) = 6$ —— (iv)

Solving (ii) and (iii) $12 + 2f'(1) + f'(1) + 3 = 0$

$3f'(1) = -15 \Rightarrow f'(1) = -5$

from (iii) $f''(2) = 12 + 2(-5) = 2$

$\therefore$ from (i) $f(1) = 1 + f'(1) + f''(2) + f'''(3)$

$= 1 - 5 + 2 + 6 = 4$ Ans.

56. $f(x) = \max.\{4,\ 1+x^2,\ x^2-1\},\ \forall x \in R$,

Then $\lim\limits_{x\to\sqrt{3}} f(x) + \lim\limits_{x\to-\sqrt{3}} f(x)$ is equal to ——

Soln. 8. when $\lim\limits_{x\to\sqrt{3}}$ max. $f(x) = 4$

$\lim\limits_{x\to-\sqrt{3}}$ max. $f(x) = 4$

$\therefore \lim\limits_{x\to\sqrt{3}} f(x) + \lim\limits_{x\to-\sqrt{3}} f(x) = 4 + 4$

$= 8$ Ans.

Happy Reading

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