Solution Of Self Evaluation Test-01 (Paper-II)

Topics :Limit, Continuity , Differentiability and Differentiation

SECTION - I

Straight Objective Type

This section contains 6 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

20. The number of points of discontinuity of the function $f(x) = \lim_{n\to\infty} \dfrac{2\cos x}{3^n + (2\sin x)^{2n}}$ is

(A) 0 (B) 1 (C) 5 (D) infinite

Soln:

$$\lim_{n\to\infty} \frac{2\cos x}{3^n + 2^{2n}(\sin x)^{2n}}$$

$$\lim_{n\to\infty} \frac{2\cos x}{3^n + 4^n(\sin x)^{2n}}$$

$$\lim_{n\to\infty} \left(\frac{1}{4}\right)^n \frac{2\cos x}{\left(\frac{3}{4}\right)^n + (\sin^2 x)^n} = 0 \quad \text{when } x = \pi/2$$

$= \text{not defined otherwise}$

Hence, there will be infinite point of discontinuity

21. If $[x]$ denotes the greatest integer less than or equal to x then $\lim_{n\to\infty} \dfrac{[x] + [2x] + [3x] + \cdots + [nx]}{n^2}$ equals

(A) $\frac{x}{2}$ (B) $x/3$ (C) x (D) 0

Soln. using Sandwitch theorem

$$x-1 < [x] \le x$$

$$2x-1 < [2x] \le 2x$$

$$3x-1 < [3x] \le 3x$$

$$\vdots$$

$$nx-1 < [nx] \le nx$$

Adding we get,

$$(x+2x+3x+\cdots+nx) - n = [x]+[2x]+\cdots+[nx] \le x+2x+3x+\cdots+nx$$

$$\Rightarrow x(1+2+3+\cdots+n) - n < [x]+[2x]+\cdots+[nx] \le x(1+2+3+\cdots+n)$$

$$\Rightarrow x\,\frac{n(n+1)}{2} - n < [x]+[2x]+\cdots+[nx] \le x\,\frac{(n+1)n}{2}$$

$$\Rightarrow \lim_{n\to\infty} x\,\frac{\cancel{n^2}\left(1+\frac{1}{n}\right)}{2\cancel{n^2}} - \frac{\cancel{n}}{n^{\cancel{2}}} < \lim_{n\to\infty} \frac{]\ [2x]+\cdots+[nx]}{n^2} \le \lim_{n\to\infty} \frac{x\cdot\cancel{n^2}\left(1+\frac{1}{n}\right)}{\cancel{n^2}\times 2}$$

$$\Rightarrow \frac{x}{2} < \lim_{n\to\infty} \frac{[x]+[2x]+\cdots+[nx]}{n^2} \le \frac{x}{2}$$

$$\therefore \lim_{n\to\infty} \frac{[x]+[2x]+\cdots+[nx]}{n^2} = \frac{x}{2}$$ Ans

22. $\lim\limits_{x \to \infty} \dfrac{\sqrt{x^2+1} - \sqrt[3]{x^3+1}}{\sqrt[4]{x^4+1} - \sqrt[5]{x^4+1}}$ equals

(A) 1 (B) 0 (C) -1 (D) None

Soln. $\lim\limits_{x \to \infty} \dfrac{(x^2+1)^{1/2} - (x^3+1)^{1/3}}{(x^4+1)^{1/4} - (x^4+1)^{1/5}}$

$$\lim_{x \to \infty} \frac{\cancel{x}\left[\left(1+\frac{1}{x^2}\right)^{1/2} - \left(1+\frac{1}{x^3}\right)^{1/3}\right]}{\cancel{x}\left[\left(1+\frac{1}{x^4}\right)^{1/4} - \left(\frac{1}{x}+\frac{1}{x^5}\right)^{1/5}\right]}$$

$$\lim_{x \to \infty} \frac{1-1}{1-0} = 0 \quad \underline{\text{Ans.}}$$

23. $\lim\limits_{n \to \infty} \left(\dfrac{1}{1-n^2} + \dfrac{2}{1-n^2} + \cdots + \dfrac{n}{1-n^2}\right)$ is equal to

(A) 0 (B) $-\frac{1}{2}$ (C) $\frac{1}{2}$ (D) None of these

Soln. $\lim\limits_{n \to \infty} \dfrac{1+2+3+\cdots+n}{1-n^2}$

$$\lim_{n \to \infty} \frac{n(n+1)}{2(1-n^2)} = \lim_{n \to \infty} \frac{\cancel{n^2}\left(1+\frac{1}{n}\right)}{2\cancel{n^2}\left(\frac{1}{n^2}-1\right)}$$

$$= \frac{1+0}{2(0-1)} = \frac{-1}{2} \quad \underline{\text{Ans.}}$$

24. $\lim_{x\to 0} \frac{e^{x^2} - \cos x}{x^2}$ equal to

(A) $3/2$ (B) $1/2$ (C) $2/3$ (D) None of these

Soln $\lim_{x\to 0} \frac{e^{x^2} 2x + \sin x}{2x}$ using LH Rule.

$\lim_{x\to 0} e^{x^2} + \frac{\sin x}{2x} = 1 + \frac{1}{2} = 3/2$

25. Let $f(x) = \frac{\sin(400\pi[x])}{1+[x]^2}$, where $[x]$ is greatest integer less than or equal to x; then

(A) $f(x)$ is not differentiable at some points

(B) $f'(x)$ exists but different from zero.

(C) $f'(x) = 0$ for all x

(D) $f'(x) = 0$ but f is not a constant function

Soln. $[x]$ is greatest integer function whose range is integer. $[x]\pi$ is an integral multiple of π

$\therefore \sin(400\pi[x]) = 0 \;\forall$ all x.

$\therefore f(x) = \frac{\sin(400\pi[x])}{1+[x]^2} = 0$ for all x $\therefore f'(x) = 0$

SECTION - II

Integer Answer Type

This section contains 5 questions. The answer to each of the questions is a single digit integer, ranging from 0 to 9. The appropriate bubbles below the respective question numbers in the ORS have to be darkened. For example, if the correct answers to question numbers X, Y and Z (say) are 6, 0 and 9, respectively, then the correct darkening of bubbles.

26. If $\lim_{x\to 0} \dfrac{axe^{x} - b\log_e(1+x) + cxe^{-x}}{x^2 \sin x} = 2$,

Then the numerical value of $\left(\dfrac{a+b+c}{3}\right)$ is ...

Soln.

Useful Series

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

$$\log_e(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots$$

$$e^{-x} = 1 - \frac{x}{1!} + \frac{x^2}{2!} - \cdots$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots$$

$$\lim_{x\to 0} \frac{ax\left(1+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}-\cdots\right) - b\left(x-\frac{x^2}{2}+\frac{x^3}{3}-\cdots\right) + cx\left(1-\frac{x}{1!}+\frac{x^2}{2!}-\cdots\right)}{x^2\left[x-\frac{x^3}{3!}+\frac{x^5}{5!}-\cdots\right]} = 2$$

$$\lim_{x\to 0} \frac{(a-b+c)x + \left(\frac{a}{1!}+\frac{b}{2}-\frac{c}{1!}\right)x^2 + \left(\frac{a}{2!}-\frac{b}{3}+\frac{c}{2!}\right)x^3 + \cdots}{x^3\left[1-\frac{x^2}{3!}+\frac{x^4}{5!}-\cdots\right]} = 2$$

This is possible if and only if degree of numerator and degree of Denumerator are same.

$\Rightarrow \quad a - b + c = 0 \quad \Rightarrow \quad b = a + c$ —— (i)

$\frac{a}{1!} + \frac{b}{2!} - \frac{c}{1!} = 0 \quad \Rightarrow \quad b = 2(c - a)$ —— (ii)

and $\quad \frac{a}{2} - \frac{b}{3} + \frac{c}{2} = 2 \Rightarrow \frac{a+c}{2} - \frac{b}{3} = 2$ —— (iii)

from (i) and (iii) $\quad \frac{b}{2} - \frac{b}{3} = 2 \Rightarrow \frac{b}{6} = 2$

$\Rightarrow b = 12$

from (i) and (ii) $\quad a + c = 12$

$c - a = 6 \Rightarrow 2c = 18$

$c = 9$

and $a = 3$

$\therefore \quad \frac{a+b+c}{3} = \frac{12+9+3}{3} = 8$ Ans.

27. $\lim_{x \to -1^+} \frac{\sqrt{2\pi} - \sqrt{2\pi \cos^{-1} x}}{\sqrt{x+1}} = A$, then the value of A is.

Sol^n Using L-H Rule as it is in form of $\left(\frac{0}{0}\right)$

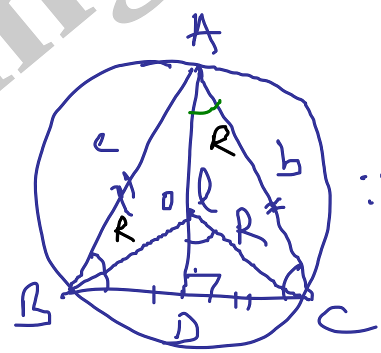
$$\lim_{x \to -1^+} \frac{0 - \sqrt{2\pi}\, \frac{1}{\cancel{2}\sqrt{\cos^{-1} x}} \times \frac{-1}{\sqrt{1-x^2}}}{\frac{1}{\cancel{2}\sqrt{x+1}}}$$

$$\lim_{x \to -1^+} \frac{\sqrt{2\pi}}{\sqrt{\cos^{-1} x}} \times \frac{1}{\sqrt{1-x}} = \frac{\sqrt{2\pi}}{\sqrt{\pi}} \times \frac{1}{\sqrt{2}}$$

$= 1$ Ans.

27. ABC is an isosceles triangle inscribed in a circle of radius R. AB = AC and l is a altitude from A to BC. If the triangle ABC has perimeter P and area Δ, then $1024\, R \frac{\Delta}{P^3}$ is equal to ______

Soln. Let O be the centre.



$\because b = c$

$OA = OB = OC = R$

$\because AB = AC$ and $AD \perp BC$

$\therefore AD$ divides BC i.e $BD = DC$ and $OD = l - R$

$\therefore BC = 2BD = 2\sqrt{R^2 - (l-R)^2} = 2\sqrt{2lR - l^2}$

Area of $\Delta = \frac{1}{2} \times BC \times l = l\sqrt{2lR - l^2}$

$AB = \sqrt{BD^2 + AD^2} = \sqrt{2lR - \cancel{l^2} + \cancel{l^2}} = \sqrt{2lR}$

$\therefore$ Perimeter $P = 2AB + BC = 2\left(\sqrt{2lR} + \sqrt{2lR - l^2}\right)$

$$\therefore \lim_{l \to 0} \frac{1024\, R \cdot \Delta}{P^3} = \lim_{l \to 0} \frac{1024 \times R\, l\sqrt{2lR - l^2}}{8\left(\sqrt{2lR} + \sqrt{2lR - l^2}\right)^3}.$$

$$= \lim_{l \to 0} \frac{128\, \cancel{R^{3/2}}\, \cancel{l\sqrt{l}} \left(\sqrt{2 - \frac{l}{R}}\right)}{\cancel{l\sqrt{l}}\, \cancel{R^{3/2}} \left(\sqrt{2} + \sqrt{2 - \frac{l}{R}}\right)^3} = \frac{128\sqrt{2}}{(2\sqrt{2})^3}$$

$$= \frac{128\cancel{\sqrt{2}}}{16\cancel{\sqrt{2}}} = 8 \text{ Ans.}$$

29. If $\lim_{x \to -\infty} \left(\frac{x^4 \sin \frac{1}{x} + x^2}{1 + |x|^3} \right) = k$, then $|k|$ is equal to - -

<u>Soln</u> Put $x = -\frac{1}{y}$ when $x \to -\infty$, $y = 0$

$$\lim_{y \to 0} \left(\frac{\frac{1}{y^4} \sin y + \frac{1}{y^2}}{1 - \frac{1}{y^3}} \right) = k$$

$$\lim_{y \to 0} \frac{\cancel{\frac{1}{y^3}} \left[\frac{\sin y}{y} + y \right]}{\cancel{\frac{1}{y^3}} \left[y^3 - 1 \right]} = k$$

$$\lim_{y \to 0} \frac{1 + 0}{0 - 1} = k \Rightarrow k = -1$$

$\therefore |k| = 1$

30. $\lim_{h \to 0} \frac{-\log(1+2h) + 2\log(1+h)}{h^2}$ is equal to ———

<u>Soln.</u>
$$\lim_{h \to 0} \frac{-\frac{1}{1+2h} \times 2 + \frac{2}{1+h}}{2h}$$ using LH Rule.

$$\lim_{h \to 0} \frac{\frac{1}{1+h} - \frac{1}{1+2h}}{h} = \lim_{h \to 0} \frac{\cancel{1} + 2h - \cancel{1} - h}{h(1+h)(1+2h)}$$

$$\lim_{h\to 0} \frac{\cancel{h}}{\cancel{h}(1+h)(1+2h)} = \frac{1}{(1+0)(1+0)} = 1$$

SECTION-III

Linked Comprehension Type

This section contains 1 paragraph. Based upon this paragraph, 3 multiple choice questions have to be answered. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONLY ONE** is correct.

Paragraph for question No. 31 to 33

Inscribed in a circle of radius R is a square, a circle is inscribed in a square, a new square in the circle and so on for n times

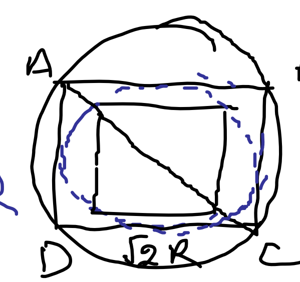
31. Sum of areas of all the circle is

(A) $4\pi R^2\left(1-\frac{1}{2^n}\right)$ (B) $2\pi R^2\left(1-\frac{1}{2^n}\right)$

(C) $3\pi R^2\left(1-\frac{1}{3^n}\right)$ (D) Involve an A.P.

Soln. diameter of circle $AC = 2R$

$\therefore$ Side of Square $ABCD = \sqrt{2}R$



= Diameter of internal circle inside ABCD

$\therefore$ Radius of internal circle $= \frac{\sqrt{2}R}{2} = \frac{R}{\sqrt{2}}$

i.e every internal circle will be $\frac{1}{\sqrt{2}}$ times of previous one.

$\therefore$ Total Area of circles are given by

$$= \pi R^2 + \pi\left(\frac{R}{\sqrt{2}}\right)^2 + \cdots \text{ n terms}$$

$$= \pi R^2 \left[1 + \frac{1}{2} + \cdots \text{ n terms}\right)$$

$$= \pi R^2 \left(\frac{1-\left(\frac{1}{2}\right)^n}{1-\frac{1}{2}}\right) = 2\pi R^2\left[1-\frac{1}{2^n}\right]$$ Ans:

32. Sum of areas of all squares as $n \to \infty$ is

(A) $2R^2$ (B) $3R^2$ (C) $4R^2$ (D) $8R^2$

Soln. $\because$ Side of Square ABCD $= \sqrt{2}R$

diameter of circle inscribed is $\sqrt{2}R$

$=$ diagonal of square inscribed

$\therefore$ Side of square $= \frac{\sqrt{2}R}{\sqrt{2}} = R$

and so on. every succeeding square has side $\frac{1}{\sqrt{2}}$ times of preceding.

$\therefore$ Total Area of all squares

$$= (\sqrt{2}R)^2 + R^2 + \left(\frac{R}{\sqrt{2}}\right)^2 + \cdots \infty$$

$$= R^2\left[2 + 1 + \frac{1}{2} + \cdots \infty\right]$$

$$= R^2\left(\frac{2}{1-\frac{1}{2}}\right) = 4R^2$$

33. The limit of the Sum of areas all circles as $n \to \infty$

(A) $2\pi R^2$ (B) $3\pi R^2$ (C) $4\pi R^2$ (D) $8\pi R^2$

Soln. Sum of all circles upto n circles

$$= 2\pi R^2 \left[1 - \frac{1}{2^n}\right]$$

when $n \to \infty$, $\frac{1}{2^n} \to 0$ $\therefore$ Required Sum $= 2\pi R^2$

Paragraph for Question Nos 34 to 36.

If f, g and h are functions having a common domain D and $h(x) \le f(x) \le g(x)$, $x \in D$ and if $\lim\limits_{x \to a} h(x) = \lim\limits_{x \to a} g(x) = l$, then $\lim\limits_{x \to a} f(x) = l$

This is known as Sandwich Theorem. using this result, compute the following limits.

34 The value of limit $\lim\limits_{x \to 0} \dfrac{|x|}{\sqrt{x^4 + 4x^2 + 7}}$

(A) 1 (B) 0 (C) $\frac{1}{2}$ (D) does not exist

Soln. $\lim\limits_{x \to 0} \dfrac{-x}{\sqrt{x^4+4x^2+7}} \le \lim\limits_{x \to 0} \dfrac{|x|}{\sqrt{x^4+4x^2+7}} \le \lim\limits_{x \to 0} \dfrac{x}{\sqrt{x^4+4x^2+7}}$

$$0 \le \lim_{x \to 0} \frac{|x|}{\sqrt{x^4+4x^2-7}} \le 0$$

$\therefore \lim_{x\to 0} \dfrac{|x|}{\sqrt{x^4+4x^2+7}} = 0$

35. $\lim_{x\to 0} x^4 \sin\left(\dfrac{1}{3\sqrt{x}}\right)$ is

Since $-1 \le \sin \dfrac{1}{3\sqrt{x}} \le 1$

$\therefore \lim_{x\to 0} x^4 \le \lim_{x\to 0} x^4 \sin\left(\dfrac{1}{3\sqrt{x}}\right) \le \lim_{x\to 0} x^4$

$\therefore \lim_{x\to 0} x^4 \sin \dfrac{1}{3\sqrt{x}} = 0$

36. Let $f(x) = x^2 \dfrac{e^{1/x} - e^{-1/x}}{e^{1/x} + e^{-1/x}}$, $x \neq 0$ and $f(0) = 1$

Then

(A) $\lim_{x\to 0^+} f(x)$ does not exist

(B) $\lim_{x\to 0} f(x)$ does not exists

(C) $\lim_{x\to 0} f(x)$ exist

(D) f is a continuous function

Soln. $\lim_{h\to 0} f(0+h) = \lim_{h\to 0} h^2 \dfrac{e^{\frac{1}{h}} - e^{-\frac{1}{h}}}{e^{1/h} + e^{-1/h}}$

$= \lim_{h\to 0} h^2 \dfrac{1 - e^{-\frac{1}{2h}}}{1 + e^{-\frac{1}{2h}}} = 0 \times \dfrac{1-0}{1+0} = 0$

$$\lim_{h\to 0} f(0-h) = \lim_{h\to 0} h^2 \frac{e^{-\frac{1}{h}} - e^{\frac{1}{h}}}{e^{-\frac{1}{h}} + e^{+\frac{1}{h}}}$$

$$= \lim_{h\to 0} h^2 \left(\frac{e^{-\frac{2}{h}} - 1}{e^{-\frac{2}{h}} + 1} \right) = 0 \; \frac{(0-1)}{0+1} = 0$$

$\because \lim_{h\to 0} f(0+h) = \lim_{h\to 0} f(0-h) \quad \therefore f(x)$ exist

Ans (C)

SECTION-IV

Matrix-Match Type

This section contains 2 questions. Each question contains statements given in two columns which have to be matched. The statements in **Column I** are labelled A, B, C and D, while the statements in **Column II** are labelled p, q, r, s and t. Any given statement in **Column I** can have correct matching with **One OR More** statement(s) in **Column II**. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following example. If the correct matches are A-p, s and t; B-q and r; C-p and q; and D-s and t;

37. let $f(x) = \begin{cases} [x], & -2 \le x < 0 \\ |x|, & 0 \le x \le 2. \end{cases}$ where [ ] denotes the greatest integer function

$g(x) = \sec x \;, \; x \in R - (2n+1)\frac{\pi}{2}$

Match the following statement in column I with their values in column II in the interval $\left(-\frac{3\pi}{2}, \frac{3\pi}{2}\right)$.

Column I	Column II
(A) limit of fog exists at	(p) -1
(B) limit of gof does not exist at	(q) π
(C) points of discontinuity is/are	(r) $\frac{5\pi}{6}$
(D) points of differentiability of fog is/are	(s) $-\pi$
	(t) 0

Soln. $fog(x) = f(g(x)) = f(\sec x) = [\sec x]$, $-2 \le \sec x \le 1$

(A)

$= |\sec x|$, $1 \le \sec x \le 2$

Clearly, fog exists at all values of x

i.e $x = -1, \pi, \frac{5\pi}{6}, -\pi, 0 \in \left(-\frac{3\pi}{2}, \frac{3\pi}{2}\right)$

$\therefore$ p, q, r, s, t all are true.

(B) $gof(x) = g([x]) = \sec[x]$, $-2 \le x < 0$

$= g(|x|) = \sec|x|$, $0 \le x \le 2$

$\therefore$ gof (x) exists for all integral value of x.

Hence, it exists at $x = 0, -1$

$\therefore$ p, t are Answer.

(C) $\because fog(x) = [\sec x]$, $-2 \le \sec x < -1$

$= |\sec x|$, $1 \le \sec x \le 2$

$\lim_{h \to 0} fog(\pi + h) = \lim_{h \to 0} [\sec(\pi + h)] = -2$

$\lim_{h \to 0} fog(\pi - h) = \lim_{h \to 0} [\sec(\pi - h)] = -1$

Hence, fog (x) is discontinuous at π

Similarly, it discontinuous at $-\pi$ $\therefore$ Ans q, s

(D) $\because$ fog are not continuous at π and $-\pi$

Hence, it is not differentiable at $x = \pi$ and $-\pi$

and it is differentiable at all value $-1, 0$

and $\frac{5\pi}{6}$. Ans p, q, t

38. If a is an Integer

Column I	Column II
(A) $\lim_{x\to a} \frac{x-a}{\sin \pi x}$	(p) $\frac{(-1)^a \pi}{6}$
(B) $\lim_{x\to a} \frac{1}{x-a}\left[\frac{1}{\sin \pi x} - \frac{(-1)^a}{(x-a)\pi}\right]$	(q) $(-1)^a \frac{3}{\pi}$
(C) $\lim_{x\to \frac{\pi}{6}} \frac{\sin(a\pi + x)}{x}$	(r) $\frac{(-1)^a}{\pi}$
(D) The integer a for which $\lim_{x\to 0} \frac{(4\cos x - 4)(11\cos x - 11e^x)}{x^a}$ is a finite non zero number	(s) 3
	(t) = 2

Soln. (A) $\lim_{x\to a} \frac{x-a}{\sin \pi x}$ $\left(\frac{0}{0}\right)$

$\therefore$ By LH Rule

$\lim_{x\to a} \frac{1-0}{\pi \cos \pi x} = \frac{1}{\pi \cos \pi a} = \frac{1}{\pi}(-1)^a.$

$\therefore$ Hence answer is (R_2)

(B) $\lim_{x\to a} \frac{1}{x-a}\left[\frac{1}{\sin \pi x} - \frac{(-1)^a}{(x-a)\pi}\right]$

put $x = a+h$

when $x \to a$, $h \to 0$

$\lim_{h\to 0} \frac{1}{h}\left[\frac{1}{\sin \pi(a+h)} - \frac{(-1)^a}{(\cancel{a}+h-\cancel{a})\pi}\right]$

$\lim_{h\to 0} \frac{1}{h}\left[\frac{(-1)^a}{\sin(\pi h)} - \frac{(-1)^a}{\pi h}\right]$

$\lim_{h\to 0} \frac{(-1)^a}{h}\left[\frac{1}{\sin \pi h} - \frac{1}{\pi h}\right]$

$\lim_{h\to 0} \frac{(-1)^a}{h} \frac{\pi h - \sin \pi h}{\pi h \sin \pi h}$ $\left(\frac{0}{0}\right)$

By LH Rule

$\lim_{h\to 0} (-1)^a \frac{\pi - \pi \cos \pi h}{\pi\left[2h \sin \pi h + h^2 (\cos \pi h)\pi\right]}$

$$\lim_{h\to 0} (-1)^a \frac{1-\cos\pi h}{\left[2h\sin\pi h + \pi h^2\cos\pi h\right]} \left(\frac{0}{0}\right)$$

$$\lim_{h\to 0} (-1)^a \frac{2\sin^2\frac{\pi h}{2}}{h^2\left[2\pi\frac{\sin\pi h}{\pi h} + \pi\cos\pi h\right]}$$

$$\lim_{h\to 0} (-1)^a \frac{\cancel{2}\left(\frac{\sin\frac{\pi h}{2}}{\frac{\pi h}{2}}\right)^2 \times \frac{\pi^2 \cancel{h^2}}{\cancel{4}\,2}}{\cancel{h^2}\left[2\pi + \pi\cos\pi h\right]}$$

$$\lim_{h\to 0} (-1)^a \frac{\frac{\pi^2}{2}}{2\pi+\pi} = (-1)^a \frac{\pi^2/2}{3\pi} = \frac{(-1)^a\pi}{6}$$

(C) $$\lim_{x\to\frac{\pi}{6}} \frac{\sin(a\pi+x)}{x} = \lim_{x\to\frac{\pi}{6}} (-1)^a \frac{\sin x}{x}$$

$$= (-1)^a \frac{\frac{1}{2}}{\frac{\pi}{6}} = (-1)^a \frac{3}{\pi}$$

$\therefore$ Ans (q)

(D) $$\lim_{x\to 0} 4\, \frac{(\cos x - 1)\,11\,(\cos x - e^x)}{x^a}$$

$$\lim_{x\to 0} \frac{44\left[\cancel{1}-\frac{x^2}{2!}+\frac{x^4}{4!}-\cdots-\cancel{1}\right]\left[\cancel{1}-\frac{x^2}{2!}+\frac{x^4}{4!}-\cdots-\left(\cancel{1}+\frac{x}{1!}+\frac{x^2}{2!}+\cdots\right)\right]}{x^a}$$

$$\lim_{x\to 0} \frac{44\,x^2\left[-\frac{1}{2!}+\frac{x^2}{4!}-\cdots\right]x\left[-\frac{1}{1!}-x+\cdots\right]}{x^a}$$

$$\lim_{x\to 0} \frac{44\left[-\frac{1}{2!}+\frac{x^2}{4}-\cdots\right]\left(-\frac{1}{1!}-x+\cdots\right]}{x^{a-3}}$$

is finite and non zero

if $a-3=0 \Rightarrow a=3$

Ans (S).