Solution Of Test Paper For IIT-JEE

For Class XI Students (Sets, Relation & Functions)

SECTION-I (only one correct answer)

21. Let A be a non empty set. If all possible number of subset of $A \times A$ is 512, then the cardinal number of A is

(A) 9 (B) 8 (C) 3 (D) 2

Soln: (C) We know total number of subset in a set $= 2^n$.

$\therefore 2^n = 512 = 2^9$ $\therefore$ no. of elements in $A \times A = 9$

Also, we know that $n(A \times B) = n(A) \times n(B)$

$\therefore n(A \times A) = n(A) \times n(A)$

$\Rightarrow 9 = n(A) \times n(A) \Rightarrow n(A) = 3$ Ans.

22. If $n(A \cap B) = 5$, then $n(A \times B) \cap (B \times A)$ is equal to

(A) 5 (B) 25 (C) 0 (D) 125

Soln: (B) $\because n(A \cap B) = 5$, Then no. of common elements in $n(A \times B) = 25$ and also in $n(B \times A) = 25$

$\therefore$ In $n(A \times B) \cap n(B \times A) = 25$

23. In right angle triangle ABC, the value of $\sin^2 A + \sin^2 B + \sin^2 C$ is

(A) 1 (B) 0 (C) 2 (D) -1

Soln: (C) Let ABC is right angle triangle, $\angle C = 90^\circ$

$\therefore A + B = 90^\circ \Rightarrow B = 90^\circ - A \Rightarrow \sin^2 B = \sin^2(90^\circ - A)$
$= \cos^2 A$

$\therefore \sin^2 A + \sin^2 B + \sin^2 C$

$= \sin^2 A + \cos^2 A + \sin^2 90^\circ$

$= 1 + 1 = 2$ Ans.

24. In ΔABC, $\angle C = \frac{\pi}{2}$. If $\tan\frac{A}{2}$ and $\tan\frac{B}{2}$ are the roots of the equation $px^2+qx+r=0$ $(p \neq 0)$, then

(A) $p+q=r$ (B) $q+r=p$

(C) $p+r=q$ (D) $q=r$

Soln. (A). $\tan\frac{A}{2} + \tan\frac{B}{2} = -\frac{q}{p}$ $\left(\because \text{Sum of roots} = -\frac{b}{a}\right)$

$\tan\frac{A}{2}\tan\frac{B}{2} = \frac{r}{p}$ $\left(\because \text{Product of roots} = \frac{c}{a}\right)$

$\because \angle C = 90^\circ \therefore A+B = 90^\circ \Rightarrow \frac{A}{2}+\frac{B}{2} = 45^\circ$

Now,

$$\tan\left(\frac{A}{2}+\frac{B}{2}\right) = \tan 45^\circ$$

$$\frac{\tan\frac{A}{2}+\tan\frac{B}{2}}{1-\tan\frac{A}{2}\tan\frac{B}{2}} = 1 \Rightarrow \frac{-\frac{q}{p}}{1-\frac{r}{p}} = 1$$

$$\Rightarrow -\frac{q}{p} = 1-\frac{r}{p} \Rightarrow \frac{r}{p} = \frac{q}{p}+1$$

$$\Rightarrow r = q+p$$

25. Let $f(x) = \sqrt{9-x^2}$, such that $D_f = [a,b]$, then the length of the interval $[a,b]$ is

(A) 0 (B) 4 (C) 6 (D) 2

Soln: (C) $9-x^2 \geq 0 \Rightarrow x^2-9 \leq 0$

$\Rightarrow (x-3)(x+3) \leq 0 \Rightarrow x \in [-3,3]$

$\therefore$ length of Interval in $[a,b]$ is $b-a = 3-(-3)$
$= 6$

26. Statement :-1. A function $f: R \to R$ is defined as $f(x) = x+5$. If $f(1) = 6$, then 6 is called the image of 1 under the function f.

Statement : 2 – The set of all images is called the range of the function

Statement : 3 – Range of the function is a subset of domain.

Soln. (B) $\because f(x) = x+5$

$\therefore f(1) = 1+5 = 6$ $\therefore$ 6 is image of 1. Hence Statement 1 is true.

Range is the set of all elements of B in $f: A \to B$ which has at least one pre image. Hence, Statement 2 is also correct. While Range is subset of codomain. Hence Statement 3 is false. Hence answer TTF.

Section -B (Multiple Correct Answer Type)

27. Let $y = f(x) = \dfrac{x+2}{x-1}$, then

(A) $D_f = R - \{1\}$ (B) f is a rational function of x.

(B) $x = f(y)$ (D) $f(1) = 3$

Soln. Clearly function is not defined at $x=1$, $\therefore$ domain $R-\{1\}$ and $f(x)$ is a rational function.

$\therefore y = \dfrac{x+2}{x-1} \Rightarrow xy - y = x+2 \Rightarrow (y-1)x = 2+y$

$x = \dfrac{2+y}{y-1} = f(y)$

Hence, A, B, C are correct.

28 Let A be a non empty set such that $A = \{1, 2, 3\}$. Then

(A) $n(A) = 3$ (B) Number of subsets of $A = 8$

(C) $n(A \times A) = 9$ (D) Number of relation on A is 512

Soln. $\because$ There are only three elements in set A $\therefore n(A) = 3$

Total no. of subset in a set of n elements $= 2^n$.

$\therefore$ no. of subset of A is $= 2^3 = 8$

$n(A\times A) = n(A) \times n(A) = 3 \times 3 = 9$

Number relation from set A to B is $= 2^{mn}$.

$\therefore$ no. of relation in set A is $= 2^9 = 512$

$\therefore$ A, B, C, D all are correct.

29. If $f(x) = \cos[\pi]x + \cos[-\pi]x$, where [ ] is G.I.F, then

(A) $f(\pi) = 0$ (B) $f(\frac{\pi}{2}) = 1$

(C) $f(-\pi) = 0$ (D) $f(\frac{\pi}{4}) = -1 - \frac{1}{\sqrt{2}}$

Soln.

$f(x) = \cos 3x + \cos 4x$ $\quad \because [\pi] = 3$, $[-\pi] = -4$

$\therefore f(\pi) = \cos 3\pi + \cos 4\pi$
$= -1 + 1 = 0$

$f(\frac{\pi}{2}) = \cos 3\frac{\pi}{2} + \cos 4 \cdot \frac{\pi}{2} = 0 + 1 = 1$

$f(-\pi) = \cos 3\pi + \cos 4\pi = -1 + 1 = 0$

$f(\frac{\pi}{4}) = \cos 3\frac{\pi}{4} + \cos \cancel{4} \cdot \frac{\pi}{\cancel{4}} = -\frac{1}{\sqrt{2}} - 1$

Hence all above are correct.

30. ABC is a triangle such that
$\sin(2A+B) = \sin(C-A) = -\sin(B+2C) = \frac{1}{2}$
If A, B, C are in A.P. Then

(A) $\angle A = 45^\circ$ (B) $\angle B = 60^\circ$

(C) $\angle C = 75^\circ$ (D) $\angle A - \angle B = 25^\circ$

Soln.

$\because$ A, B, C are in A.P.

$\therefore A = B - d$ where d is common difference.
$C = B + d$

$\therefore$ Now, $A + B + C = 180^\circ$ (Sum of all angles in a Δ)
$B - d + B + B + d = 180^\circ \Rightarrow 3B = 180 \Rightarrow B = 60^\circ$

$\because \sin(2A+B) = \frac{1}{2} \Rightarrow 2A+B = 150^\circ$

$\Rightarrow 2A + 60 = 150 \Rightarrow 2A = 90^\circ$

$\Rightarrow A = 45^\circ$

$\therefore \angle C = 180^\circ - (A+B) = 180 - (45+60) = 75^\circ$

$\therefore \angle A - \angle B = 45 - 60 = -15^\circ \quad \therefore D$ is not hold.

But A, B, C are Correct.

Section-3 Reasoning Type

31. Let $f(x) = 2\sin 3x$

Statement : 1- The period of $f(x)$ is 2π.

Statement : 2- $f(x+2\pi) = f(x)$

Soln. (D) Period of $\sin x$ is 2π

$\therefore$ Period of $2\sin 3x$ is $\frac{2\pi}{3}$

$\therefore$ Statement I is False, while statement II is true.

32. In acute angle triangle ABC, $a > b > c$

Statement 1:- $r_1 > r_2 > r_3$

Statement 2:- $\cos A < \cos B < \cos C$

Soln. (A) We know, $r_1 = \frac{\Delta}{s-a}$, $r_2 = \frac{\Delta}{s-b}$, $r_3 = \frac{\Delta}{s-c}$

$\because a > b > c \Rightarrow s-a < s-b < s-c$

$\Rightarrow \frac{\Delta}{s-a} > \frac{\Delta}{s-b} > \frac{\Delta}{s-c} \Rightarrow r_1 > r_2 > r_3$

Also, $\because a > b > c \Rightarrow \cos A > \cos B > \cos C$

Section-IV (Paragraph Type)

Let $a, b \in R^+$, Then $A.M. = \frac{a+b}{2}$ and $G.M = \sqrt{ab}$

Now, $A.M - G.M = \frac{a+b}{2} - \sqrt{ab}$

$$= \frac{1}{2}\left(a+b-2\sqrt{ab}\right)$$

$$= \frac{1}{2}\left(\sqrt{a}-\sqrt{b}\right)^2 \geq 0$$

$\Rightarrow AM - G.M \geq 0$

$\Rightarrow AM \geq GM$ and equality hold when $a = b$

Use this Concept and answer the following questions

33. Let $f(x) = x^2 + \frac{2}{x}$, $x > 0$, Then range of $f(x)$ is

(A) $[0, \infty)$ (B) $[1, \infty)$

(C) $[3, \infty)$ (D) $(-\infty, -2]$

Soln. for, a, b, c $\because AM \geq GM \Rightarrow \frac{a+b+c}{3} \geq \sqrt[3]{abc}$ — ①

(C) $\left[\because x^2 + \frac{2}{x} = x^2 + \frac{1}{x} + \frac{1}{x}\right]$

using ①, $\frac{x^2 + \frac{1}{x} + \frac{1}{x}}{3} \geq \sqrt[3]{x^2 \times \frac{1}{x} \times \frac{1}{x}}$.

$$x^2 + \frac{1}{x} + \frac{1}{x} \geq 3 \times 1$$

$x^2 + \frac{2}{x} \geq 3$ $\therefore$ Range $[3, \infty)$

34. Let $f(x) = \sin^2 x + \csc^2 x$, $0 < x \leq \frac{\pi}{2}$, Then the range of $f(x)$ is

(A) $[6, \infty)$ (B) $[1, \infty)$ (C) $[4, \infty)$ (D) $[2, \infty)$

Soln: (D) $AM \geq GM$

$$\Rightarrow \frac{\sin^2 x + \csc^2 x}{2} \geq \sqrt{\sin^2 x \times \csc^2 x}$$

$$\Rightarrow \sin^2 x + \csc^2 x \geq 2 \quad \therefore \text{Range } [2, \infty)$$

35. Let $f(x) = x + x^2 + \frac{1}{x} + \frac{1}{x^2}$, $x > 0$ then the range of $f(x)$ is

(A) $[1, \infty)$ (B) $[2, \infty)$

(C) $[4, \infty)$ (D) $[6, \infty)$

Soln (C) $\because AM \geq GM$

$$\therefore \frac{x + x^2 + \frac{1}{x} + \frac{1}{x^2}}{4} \geq \sqrt[4]{\cancel{x} \times \cancel{x^2} \times \frac{1}{\cancel{x}} \times \frac{1}{\cancel{x^2}}}$$

$$\Rightarrow x + x^2 + \frac{1}{x} + \frac{1}{x^2} \geq 4 \quad \therefore \text{Range } [4, \infty)$$

Section-V (Matrix

36. Match the following columns

Column I	Column II
(A) In any $\triangle ABC$, $a\cos B + b\cos A$	(p) $a+c$
(B) In any $\triangle ABC$, $(b+a)\cos C + c(\cos A + \cos B)$	(q) $b+c$
(C) In any $\triangle ABC$, $(a+c)\cos B + b(\cos A + \cos C)$	(r) $2a$
(D) In any $\triangle ABC$, $(b+c)\cos A + a(\cos B + \cos C)$	(s) c
	(t) $a+b$

$A \to s$, $B \to t$, $C \to p$, $D \to q$

Soln: (A) $a\cos B + b\cos A = c$ (projection formula)

(B) $(b\cos C + c\cos A) + (a\cos C + c\cos B) = a + b$

(C) $(a\cos B + b\cos A) + (c\cos B + b\cos C) = c + a$

(D) $(b\cos A + a\cos B) + (c\cos A + a\cos C) = c + b$

37. Match the following Column

Column I	Column II
(A) Let $f(x) = \dfrac{x+3}{x^2-4}$, Then the domain of $f(x)$ is	(p) $[0, \infty)$
(B) Let $f(x) = x^2+1$, Then the range of $f(x)$ is	(q) $(-\infty, \infty)$
(C) Let $f(x) = \sqrt{x-2}$, Then the range of $f(x)$ is	(r) $[2, \infty)$
(D) Let $f(x) = x^{2011} + x^{2000} + 1$ Then the domain of $f(x)$ is	(s) $[1, \infty)$
	(t) $R - \{-2, 2\}$

Soln. $A \to t$, $B \to s$, $C \to p$, $D \to q$

Explanation:- (A) $f(x)$ is defined for all x except $x^2 - 4 = 0$ i.e $x = \pm 2$

$\therefore$ domain $R - \{-2, 2\}$

(B) $\because x^2 \geq 0 \Rightarrow x^2 + 1 \geq 1$

$\therefore$ Range of $f(x) = [1, \infty)$

(C) $f(x) = \sqrt{x-2}$ is defined for all $x - 2 \geq 0$

$x \geq 2$

$\therefore$ Range will be given by $[0, \infty)$

(D) $\because$ it is algebraic Polynomial, Hence defined for all real x, $\therefore$ domain is $(-\infty, \infty)$

Section-VI (Integer Answer Type)

38. In a Δ ABC, the value of

$$\frac{b^2-c^2}{a^2}\sin 2A + \frac{c^2-a^2}{b^2}\sin 2B + \frac{a^2-b^2}{c^2}\sin 2C \text{ is } \_\_\_$$

Soln: (0) We know, $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ (Sine formulae)

$\therefore$ $a = 2R\sin A$, $b = 2R\sin B$, $c = 2R\sin C$

$$\therefore \frac{b^2-c^2}{a^2}\sin 2A = \frac{4R^2\sin^2 B - 4R^2\sin^2 C}{4R^2\sin^2 A}\sin 2A$$

$$= \frac{\sin^2 B - \sin^2 C}{\sin^2 A} \times 2\sin A\cos A$$

$$= \frac{\sin(B+C)\sin(B-C)\,2\cos A}{\sin A}$$

$$= \frac{2\sin(\pi-A)\sin(B-C)\cos A}{\sin A}$$

$$= \frac{2\cancel{\sin A}\sin(B-C)\cos\{\pi-(A+B)\}}{\cancel{\sin A}}$$

$$= -2\cos(B+C)\sin(B-C)$$

$$\frac{b^2-c^2}{a^2}\sin 2A = \sin 2C - \sin 2B.$$

Similarly, $\frac{c^2-a^2}{b^2}\sin 2B = \sin 2A - \sin 2C$

$$\frac{a^2-b^2}{c^2}\sin 2C = \sin 2B - \sin 2A$$

Adding, $\frac{b^2-c^2}{a^2}\sin 2A + \frac{c^2-a^2}{b^2}\sin 2B + \frac{a^2-b^2}{c^2}\sin 2C = 0$

39. If sides of $\triangle ABC$, satisfying $\cos\theta = \frac{a}{b+c}$, $\cos\phi = \frac{b}{a+c}$, $\cos\psi = \frac{c}{a+b}$, where $\theta, \phi, \psi \in (0, \frac{\pi}{2})$

Then the value of $\tan^2\frac{\theta}{2} + \tan^2\frac{\phi}{2} + \tan^2\frac{\psi}{2}$ is ____

Soln. (1) $\cos\theta = \frac{a}{b+c} \Rightarrow \dfrac{1-\tan^2\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}} = \frac{a}{b+c}$.

using componendo and dividendo

$$\Rightarrow \frac{\cancel{1}+\tan^2\frac{\theta}{2} - (\cancel{1}-\tan^2\frac{\theta}{2})}{1+\cancel{\tan^2\frac{\theta}{2}} + 1 - \cancel{\tan^2\frac{\theta}{2}}} = \frac{b+c-a}{b+c+a}$$ [$\because a+b+c = 2s$]

$$\Rightarrow \tan^2\frac{\theta}{2} = \frac{a+b+c-2a}{2s} = \frac{2s-2a}{2s} = \frac{s-a}{s}$$

Similarly, $\tan^2\frac{\phi}{2} = \frac{s-b}{s}$

$\tan^2\frac{\psi}{2} = \frac{s-c}{s}$

Adding, $\tan^2\frac{\theta}{2} + \tan^2\frac{\phi}{2} + \tan^2\frac{\psi}{2} = \frac{s-a}{s} + \frac{s-b}{s} + \frac{s-c}{s}$

$$= \frac{3s-(a+b+c)}{s} = \frac{3s-2s}{s}$$

$$= \frac{s}{s} = 1$$

40. Let ABC is a triangle whose area is 12 sq. cm. and base (BC) is 6 cm. If the difference of base angle is 60°. Then the value of $8\sin A - 6\cos A$ is ____

Soln. 3.