SOLVED EXAMPLES

Example – 1

Let ABC and PQR be two triangles in a plane. Assume that the perpendiculars from the points A, B, C to the sides QR, RP, PQ respectively are concurrent. Using vector methods, prove that the perpendiculars from P, Q, R to BC, CA, AB respectively are also concurrent.

Solution:

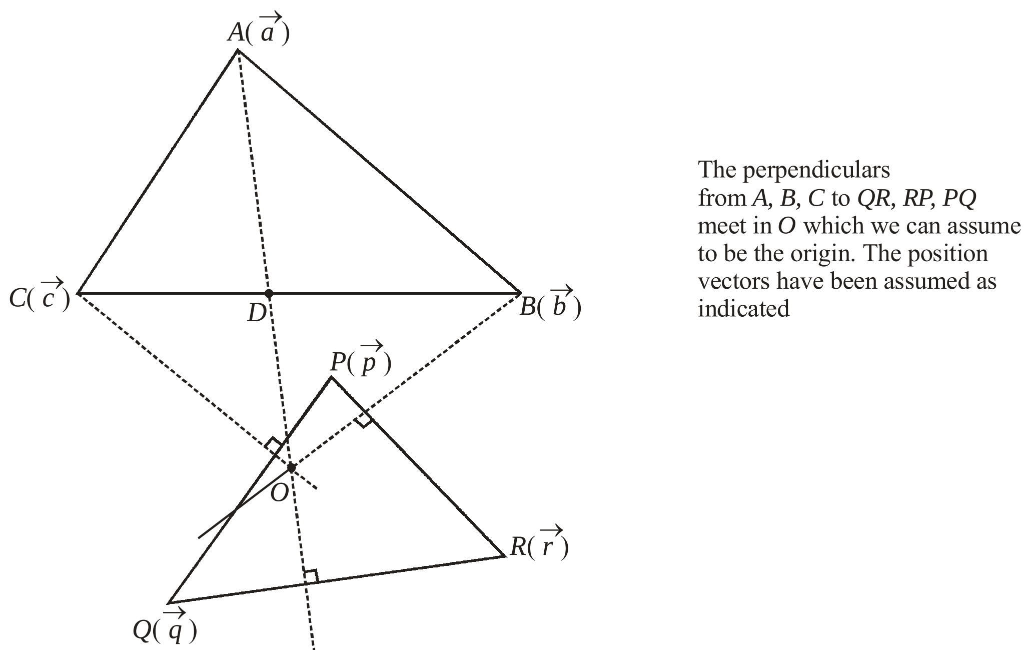

Fig - 70

We have,

$$OA \perp QR, \quad OB \perp RP, \quad OC \perp PQ$$

$$\Rightarrow \quad \vec{a}\cdot(\vec{r}-\vec{q}) = \vec{b}\cdot(\vec{p}-\vec{r}) = \vec{c}\cdot(\vec{q}-\vec{p}) = 0 \qquad ...(1)$$

We now proceed as follows. We draw the perpendiculars from P, Q to BC, CA respectively (not shown in the figure) and assume that these perpendiculars meet in $Z(\vec{z})$.

If we find $Z(\vec{z})$ and show that RZ is perpendicular to AB, the concurrency will be established.

Since, by assumption,

$$\overrightarrow{PZ} \perp \overrightarrow{BC} \text{ and } \overrightarrow{QZ} \perp \overrightarrow{CA}$$

$$\Rightarrow \quad (\vec{p}-\vec{z})\cdot(\vec{c}-\vec{b}) = 0 \text{ and } (\vec{q}-\vec{z})\cdot(\vec{a}-\vec{c}) = 0$$

$$\Rightarrow \quad \left\{\vec{p}\cdot(\vec{c}-\vec{b}) + \vec{q}\cdot(\vec{a}-\vec{c})\right\} = \vec{z}\cdot\left\{(\vec{c}-\vec{b}) + (\vec{a}-\vec{c})\right\}$$

The left hand side can be modified using (1), and thus we obtain,

$$\left\{-\vec{p}\cdot\vec{b}+\vec{q}\cdot\vec{a}+(\vec{p}-\vec{q})\cdot\vec{c}\right\}=\vec{z}\cdot\left(\vec{a}-\vec{b}\right)$$

$$\Rightarrow \quad \vec{q}\cdot\vec{a}-\vec{p}\cdot\vec{b}=\vec{z}\cdot\left(\vec{a}-\vec{b}\right)$$

$$\Rightarrow \quad \left(\vec{a}-\vec{b}\right)\cdot\vec{r}=\vec{z}\cdot\left(\vec{a}-\vec{b}\right) \qquad \text{(Again using (1))}$$

$$\Rightarrow \quad \left(\vec{a}-\vec{b}\right)\cdot(\vec{r}-\vec{z})=0$$

$\Rightarrow$ $\vec{a}-\vec{b}$ is perpendicular to $\vec{r}-\vec{z}$.

$\Rightarrow$ AB is perpendicular to *RZ*.

This establishes the concurrency of the three perpendiculars. ◀

Example – 2

For any two vectors $\vec{u}$ and $\vec{v}$, prove that

$$\left(1+|\vec{u}|^2\right)\left(1+|\vec{v}|^2\right)=(1-\vec{u}\cdot\vec{v})^2+\left|\vec{u}+\vec{v}+(\vec{u}\times\vec{v})\right|^2$$

Solution: It should be apparent that we should start with expanding the right hand side. To expand the second term in the *RHS*, we use the relation $\left|\vec{a}+\vec{b}\right|^2=|\vec{a}|^2+\left|\vec{b}\right|^2+2\left(\vec{a}\cdot\vec{b}\right)$

Thus,

$$RHS=1+(\vec{u}\cdot\vec{v})^2-2\vec{u}\cdot\vec{v}+\{\vec{u}+\vec{v}+(\vec{u}\times\vec{v})\}\cdot\{\vec{u}+\vec{v}+(\vec{u}\times\vec{v})\}$$

$$=1+(\vec{u}\cdot\vec{v})^2-2\vec{u}\cdot\vec{v}+|\vec{u}|^2+|\vec{v}|^2+2\vec{u}\cdot\vec{v}+|\vec{u}\times\vec{v}|^2$$

$$=1+|\vec{u}|^2+|\vec{v}|^2+\underline{(\vec{u}\cdot\vec{v})^2+(\vec{u}\times\vec{v})^2}$$

Assuming the angle between $\vec{u}$ and $\vec{v}$ to be θ, the underlined expression can be written simply as $|\vec{u}|^2|\vec{v}|^2\cos^2\theta+|\vec{u}|^2|\vec{v}|^2\sin^2\theta=|\vec{u}|^2|\vec{v}|^2$. Thus,

$$RHS=1+|\vec{u}|^2+|\vec{v}|^2+|\vec{u}|^2|\vec{v}|^2$$

$$=\left(1+|\vec{u}|^2\right)\left(1+|\vec{v}|^2\right)$$ ◀

Example – 3

If $\vec{a}, \vec{b}, \vec{c}$ are three non-coplanar unit vectors equally inclined to one another at an angle θ such that

$$\vec{a} \times \vec{b} + \vec{b} \times \vec{c} = p\vec{a} + q\vec{b} + r\vec{c},$$

find $\vec{p}, \vec{q}, \vec{r}$ in terms of θ. Also, show that

$$p^2 + \frac{q^2}{\cos\theta} + r^2 = 2$$

Solution: Note that

$$\vec{a} \cdot \vec{a} = \vec{b} \cdot \vec{b} = \vec{c} \cdot \vec{c} = 1; \quad \vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{c} = \vec{c} \cdot \vec{a} = \cos\theta$$

Also

$$\left[\vec{a} \quad \vec{b} \quad \vec{c}\right]^2 = \begin{vmatrix} \vec{a} \cdot \vec{a} & \vec{a} \cdot \vec{b} & \vec{a} \cdot \vec{c} \\ \vec{b} \cdot \vec{a} & \vec{b} \cdot \vec{b} & \vec{b} \cdot \vec{c} \\ \vec{c} \cdot \vec{a} & \vec{c} \cdot \vec{b} & \vec{c} \cdot \vec{c} \end{vmatrix}$$

(We'll need this value later; that's why we are evaluating it here)

$$= \begin{vmatrix} 1 & \cos\theta & \cos\theta \\ \cos\theta & 1 & \cos\theta \\ \cos\theta & \cos\theta & 1 \end{vmatrix}$$

$$= 1 - 3\cos^2\theta + 2\cos^3\theta$$

$$= (1 - \cos\theta)^2 (1 + 2\cos\theta)$$

$$\Rightarrow \quad \left[\vec{a} \quad \vec{b} \quad \vec{c}\right] = (1 - \cos\theta)\sqrt{1 + 2\cos\theta}$$

The given relation is

$$\vec{a} \times \vec{b} + \vec{b} \times \vec{c} = p\vec{a} + q\vec{b} + r\vec{c}$$

Taking the dot product on both sides successively with $\vec{a}, \vec{b}, \vec{c},$ we get the following system of equations.

$$p + q\cos\theta + r\cos\theta = \left[\vec{a} \quad \vec{b} \quad \vec{c}\right]$$

$$p\cos\theta + q + r\cos\theta = 0$$

$$p\cos\theta + q\cos\theta + r = \left[\vec{a} \quad \vec{b} \quad \vec{c}\right]$$

Using Cramer's rule (or otherwise, by elimination), p, q, r can be evaluated. For example,

$$p = \frac{\begin{vmatrix} \left[\vec{a}\ \ \vec{b}\ \ \vec{c}\right] & \cos\theta & \cos\theta \\ 0 & 1 & \cos\theta \\ \left[\vec{a}\ \ \vec{b}\ \ \vec{c}\right] & \cos\theta & 1 \end{vmatrix}}{\begin{vmatrix} 1 & \cos\theta & \cos\theta \\ \cos\theta & 1 & \cos\theta \\ \cos\theta & \cos\theta & 1 \end{vmatrix}}$$

$$= \frac{\left[\vec{a}\ \ \vec{b}\ \ \vec{c}\right](1-\cos\theta)}{\left[\vec{a}\ \ \vec{b}\ \ \vec{c}\right]^2}$$

$$= \frac{1}{\sqrt{1+2\cos\theta}}$$

Similarly,

$$q = \frac{-2\cos\theta}{\sqrt{1+2\cos\theta}}, \quad r = \frac{1}{\sqrt{1+2\cos\theta}}$$

These values confirm that

$$p^2 + \frac{q^2}{\cos\theta} + r^2 = 2$$

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Example – 4

Let V be the volume of the parallelopiped formed by the vectors

$$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

$$\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$$

If $a_r, b_r, c_r, r = 1, 2, 3$ are non-negative numbers such that $\sum_{r=1}^{3}(a_r + b_r + c_r) = 3L,$ show that $V \le L^3.$

Solution: We have,

$$V = \left|\left[\vec{a} \quad \vec{b} \quad \vec{c}\right]\right|$$

$$= \left|\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}\right|$$

$$= \left|a_1b_2c_3 + a_2b_3c_1 + a_3b_1c_2 - \left(a_1b_3c_2 + a_2b_1c_3 + a_3b_2c_1\right)\right|$$

$$\le a_1b_2c_3 + a_2b_3c_1 + a_3b_1c_2 \qquad ...(1)$$

The last step is justified since all the $a_i's$, $b_i's$ and $c_i's$ are positive.

Now, by the *AM-GM* in equality, we have

$$27\left(a_1b_2c_3\right) \le \left(a_1 + b_2 + c_2\right)^3$$

$$27\left(a_2b_3c_1\right) \le \left(a_2 + b_3 + c_1\right)^3$$

$$27\left(a_3b_1c_2\right) \le \left(a_3 + b_1 + c_2\right)^3$$

Adding these three inequalities, we have

$$27\left(a_1b_2c_3 + a_2b_3c_1 + a_3b_1c_2\right) \le \left(a_1 + b_2 + c_3\right)^3 + \left(a_2 + b_3 + c_1\right)^3 + \left(a_3 + b_1 + c_2\right)^3 \qquad ...(2)$$

The right hand side is of the form

$$x^3 + y^3 + z^3$$

where $x, y, z \ge$ so that

$$x^3 + y^3 + z^3 \le \left(x + y + z\right)^3 \qquad ...(3)$$

Using (3) in (2), we have

$$27\left(a_1b_2c_3 + a_2b_3c_1 + a_3b_1c_2\right) \le \left(a_1 + a_2 + a_3 + b_1 + b_2 + b_3 + c_1 + c_2 + c_3\right)^3 = 27L^3 \qquad ...(4)$$

From (1) and (4), it follows that

$$V \le L^3$$

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Example – 5

Let $\vec{u}, \vec{v}, \vec{w}$ be three non-coplanar unit vectors and α, β, γ be the angles between $\vec{u}$ and $\vec{v}$, $\vec{v}$ and $\vec{w}$ and $\vec{w}$ and $\vec{u}$ respectively. Let $\vec{x}, \vec{y}, \vec{z}$ be the unit vectors along the bisectors of the angles α, β, γ respectively. Prove that

$$[\vec{x}\times\vec{y} \;\; \vec{y}\times\vec{z} \;\; \vec{z}\times\vec{x}] = \frac{1}{16}[\vec{u} \;\; \vec{v} \;\; \vec{w}]^2 \sec^2\frac{\alpha}{2}\sec^2\frac{\beta}{2}\sec^2\frac{\gamma}{2}$$

Solution: Since $\vec{u}, \vec{v}, \vec{w}$ are unit vectors, we have

$$|\vec{u}+\vec{v}| = 2\cos\frac{\alpha}{2}, \quad |\vec{v}+\vec{w}| = 2\cos\frac{\beta}{2}, \quad |\vec{w}+\vec{u}| = 2\cos\frac{\gamma}{2}$$

Now, since $\vec{x}, \vec{y}, \vec{z}$ are **unit** vectors along the bisectors of $\alpha, \beta, \gamma,$ we have

$$\vec{x} = \frac{\vec{u}+\vec{v}}{2\cos\alpha/2}, \quad \vec{y} = \frac{\vec{v}+\vec{w}}{2\cos\beta/2}, \quad \vec{z} = \frac{\vec{w}+\vec{u}}{2\cos\gamma/2}$$

$$LHS = [\vec{x}\times\vec{y} \quad \vec{y}\times\vec{z} \quad \vec{z}\times\vec{x}] = [\vec{x} \quad \vec{y} \quad \vec{z}]^2$$

$$= \frac{[\vec{u}+\vec{v} \quad \vec{v}+\vec{w} \quad \vec{w}+\vec{u}]^2}{64\cos^2\frac{\alpha}{2}\cos^2\frac{\beta}{2}\cos^2\frac{\gamma}{2}}$$

$$= \frac{\left(2[\vec{u} \quad \vec{v} \quad \vec{w}]\right)^2}{64\cos^2\frac{\alpha}{2}\cos^2\frac{\beta}{2}\cos^2\frac{\gamma}{2}}$$

$$= RHS$$

This proves the stated assertion. ◀

ASSIGNMENT

[LEVEL - I]

Q. 1 Prove that the line segment joining the mid-points of two sides of a triangle is parallel to the third side and equal to half of it.

Q. 2 A transversal cuts the sides *OL*, *OM* and diagonal *ON* of a parallelogram *OLNM* in *A*, *B* and *C* respectively. Prove that

$$\frac{OL}{OA}+\frac{OM}{OB}=\frac{ON}{OC}$$

Q. 3 A line makes angle α,β,γ and δ with the diagonals of a cube. Prove that

$$\cos^2\alpha+\cos^2\beta+\cos^2\gamma+\cos^2\delta=\frac{4}{3}$$

Q. 4 Prove that if two medians in a triangle are equal, then it must be isosceles.

Q. 5 Prove using vector methods the trigonometric relation

$$\sin(A\pm B)=\sin A\cos B+\cos A\sin B$$

Q. 6 Let $\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix}=0$. If the vectors $\vec{A}=\hat{i}+a\hat{j}+a^2\hat{k}$, $\vec{B}=\hat{i}+b\hat{j}+b^2\hat{k}$ and $\vec{C}=\hat{i}+c\hat{j}+c^2\hat{k}$ are coplanar, prove that $abc=-1$.

Q. 7 *ABCDEF* is a regular hexagon. *M* is the mid-point of *DE*, *N* is the mid-point of *AM* and *P* is the mid-point of *BC*. Write the vector $\overrightarrow{NP}$ using the vectors $\overrightarrow{AB}$ and $\overrightarrow{AF}$ as basis.

Q. 8 If $\vec{a},\vec{b},\vec{c}$ are three coplanar vectors and $\vec{a}$ and $\vec{b}$ are non-collinear, prove that $\vec{c}$ can be written as

$$\vec{c}=\frac{\Delta_1}{\Delta}\vec{a}+\frac{\Delta_2}{\Delta}\vec{b}$$

where

$$\Delta_1=\begin{vmatrix} \vec{c}\cdot\vec{a} & \vec{a}\cdot\vec{b} \\ \vec{c}\cdot\vec{b} & \vec{b}\cdot\vec{b} \end{vmatrix},\quad \Delta_2=\begin{vmatrix} \vec{a}\cdot\vec{a} & \vec{c}\cdot\vec{a} \\ \vec{a}\cdot\vec{b} & \vec{c}\cdot\vec{b} \end{vmatrix},\quad \Delta=\left\{|\vec{a}|^2|\vec{b}|^2-(\vec{a}\cdot\vec{b})^2\right\}$$

Q. 9 Prove that the mid-points of two opposite sides of a quadrilateral and the mid-points of the diagonals are the vertices of a parallelogram.

Q. 10 Find the equation of the plane passing through the line of intersection of the planes $\vec{r}\cdot\vec{n}_1=d_1$, $\vec{r}\cdot\vec{n}_2=d_2$ and parallel to the line of intersection of the planes $\vec{r}\cdot\vec{n}_3=d_3$ and $\vec{r}\cdot\vec{n}_4=d_4$.

Q. 11 Prove using vectors that the mid-point of the hypotenuse of a right angled triangle is equidistant from its vertices.

Q. 12 Prove that in a cuboid, the sum of the squares of the diagonals is equal to four times the sum of the squares of any three co-initial edges.

Q. 13 Show that each of the four faces of a tetrahedron subtends the same volume at the centroid.

Q. 14 P and Q are the mid-points of the non-parallel sides BC and AD of a trapezium $ABCD$. Show that

$$\text{area }(\Delta APD) = \text{area }(\Delta CQB)$$

Q. 15 Prove that the area Δ of a triangle ABC can be given by $\Delta = \frac{1}{2}\frac{a^2 \sin B \sin C}{\sin A}$ where the symbols have their usual meanings.

Q. 16 Prove that if the diagonals of a parallelogram are of equal lengths, then it must be a rectangle.

Q. 17 If $\vec{a}, \vec{b}, \vec{c}$ are coplanar vectors, prove that

(i) $\begin{vmatrix} \vec{a} & \vec{b} & \vec{c} \\ \vec{a}\cdot\vec{a} & \vec{a}\cdot\vec{b} & \vec{a}\cdot\vec{c} \\ \vec{b}\cdot\vec{a} & \vec{b}\cdot\vec{b} & \vec{b}\cdot\vec{c} \end{vmatrix} = 0$ (ii) $\begin{vmatrix} \vec{a}\cdot\vec{a} & \vec{a}\cdot\vec{b} & \vec{a}\cdot\vec{c} \\ \vec{b}\cdot\vec{a} & \vec{b}\cdot\vec{b} & \vec{b}\cdot\vec{c} \\ \vec{c}\cdot\vec{a} & \vec{c}\cdot\vec{b} & \vec{c}\cdot\vec{c} \end{vmatrix} = 0$

Q. 18 For any vector $\vec{r}$, prove that $\left(\vec{r} \times \hat{i}\right)^2 + \left|\vec{r} \times \hat{j}\right|^2 + \left|\vec{r} \times \hat{k}\right|^2 = 2\left|\vec{r}\right|^2$

[LEVEL - II]

Q. 19 In ΔABC, prove that $\cos 2A + \cos 2B + \cos 2C \geq \frac{-3}{2}$

Q. 20 The internal bisectors of the angles A, B and C of ΔABC meet the opposite sides in D, E and F respectively. Prove that

$$\text{area }(\Delta DEF) \leq \frac{1}{4}\text{area }(\Delta ABC)$$

Q. 21 Let $\vec{A}(t) = f_1(t)\hat{i} + f_2(t)\hat{j}$ and $\vec{B}(t) = g_1(t)\hat{i} + g_2(t)\hat{j}$, $t \in [0,1]$ where f_1, f_2, g_1, g_2 are continuous functions.

If $\vec{A}(t)$ and $\vec{B}(t)$ are non-zero vectors for all t and $\vec{A}(0) = 2\hat{i} + 3\hat{j}$, $\vec{A}(1) = 6\hat{i} + 2\hat{j}$, $\vec{B}(0) = 3\hat{i} + 2\hat{j}$ and $\vec{B}(1) = 2\hat{i} + 6\hat{j}$, then show that $\vec{A}(t)$ and $\vec{B}(t)$ are parallel for some t.

Q. 22 **(a)** Prove that the segment joining the middle points of the two non-parallel sides of a trapezium is parallel to the parallel sides and equal to half their sum.

(b) Prove that the segment joining the mid-points of the diagonals of a trapezium is parallel to the parallel sides and equal to half their difference.

Q. 23 In a triangle ABC, D, E, F are taken on BC, CA and AB respectively such that $\frac{BD}{DC}=\frac{CE}{EA}=\frac{AF}{FB}=n.$

Prove that area $(\Delta DEF)=\frac{n^2-n+1}{(n+1)^2}$ area (ΔABC)

Q. 24 Let $\vec{a},\vec{b},\vec{c}$ be three non-coplanar vectors, so that $\left[\vec{a}\ \ \vec{b}\ \ \vec{c}\right]\neq 0.$ Define the set of vectors, $\vec{a}_1,\vec{b}_1,\vec{c}_1$ as

$$\vec{a}_1=\frac{\vec{b}\times\vec{c}}{\left[\vec{a}\ \vec{b}\ \vec{c}\right]},\ \vec{b}_1=\frac{\vec{c}\times\vec{a}}{\left[\vec{a}\ \vec{b}\ \vec{c}\right]},\ \vec{c}_1=\frac{\vec{a}\times\vec{b}}{\left[\vec{a}\ \vec{b}\ \vec{c}\right]}$$

Observe that

$$\vec{a}\cdot\vec{a}_1=\vec{b}\cdot\vec{b}_1=\vec{c}\cdot\vec{c}_1=1$$

The system of vectors $\vec{a}_1,\vec{b}_1,\vec{c}_1$ is called the **reciprocal system** of the set of vectors $\vec{a},\vec{b},\vec{c}$.

(a) Prove that $\left[\vec{a}_1\ \vec{b}_1\ \vec{c}_1\right]=\frac{1}{\left[\vec{a}\ \vec{b}\ \vec{c}\right]}$ **(b)** Prove that $\vec{b}_1\times\vec{c}_1+\vec{c}_1\times\vec{a}_1+\vec{a}_1\times\vec{b}_1=\frac{\vec{a}+\vec{b}+\vec{c}}{\left[\vec{a}\ \vec{b}\ \vec{c}\right]}$

(c) Find explicitly the reciprocal system of the set of vectors $\vec{a}=\hat{i}+\hat{j}-\hat{k},\ \vec{b}=2\hat{i}-\hat{j}+3\hat{k}$ and $\vec{c}=\hat{i}+2\hat{j}-\hat{k}$ and prove that it satisfies the two properties above.

Q. 25 Points P, Q, R divide BC, CA and AB of ΔABC in the ratio 1 : 2. The segments AP, BQ and CR form the triangle XYZ. Prove that ΔABC and ΔXYZ have the same centroid.

Q. 26 Let $\vec{u}$ and $\vec{v}$ be two given non-collinear unit vectors and $\vec{w}$ be a vector such that $\vec{w}+(\vec{w}\times\vec{u})=\vec{v}.$

Prove that $\left|(\vec{u}\times\vec{v})\cdot\vec{w}\right|\leq\frac{1}{2}$

Q. 27 Three concurrent straight lines OA, OB, OC are produced to D, E, F respectively. AB and DE, BC and EF, CA and FD intersect in X, Y, Z respectively. Prove that X, Y, Z are collinear.

Q. 28 Let $OABC$ be a regular tetrahedron of side L. D is the circumcentre of ΔOAB and E is the mid-point of AC. Find DE.

Q. 29 Prove that the point of intersection of the diagonals of a trapezium lies on the line passing through the mid-points of the parallel sides.

Q. 30 Show that the angle between any edge and a face not containing that edge of a rectangular tetrahedron is $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$.

ANSWERS

TRY YOURSELF I

Ans. 1 – 40

Ans. 3 Mid point of the line segment joining the mid points of *AC* and *BD*

TRY YOURSELF II

Ans. 1 $c \in \left(-\frac{4}{3}, 0\right)$

Ans. 2 – 15 J

Ans, 7 $\pm\frac{1}{\sqrt{51}}\left(-5\hat{i} + \hat{j} + 5\hat{k}\right)$

Ans. 9 $a \in (-3,\ 2) \cup (2,\ 3)$

TRY YOURSELF III

Ans. 4 $\vec{r} = \frac{5}{3}\hat{i} + \frac{2}{3}\hat{j} + \frac{2}{3}\hat{k}$

TRY YOURSELF IV

Ans. 1 264 cubic units

Ans. 2 $\lambda = -\frac{146}{17}$

TRY YOURSELF VI

Ans. 2 $\vec{r} \cdot \left\{(\vec{\alpha} - \vec{a}) \times \vec{b}\right\} = \vec{a} \cdot \left\{(\vec{\alpha} - \vec{a}) \times \vec{b}\right\}$

Ans. 3 $\vec{r} \cdot \left(3\hat{i} + 8\hat{j} + \hat{k}\right) = 17$; $7\hat{i} - 11\hat{j} + 4\hat{k}$

Ans. 10 $A \equiv 2\hat{i} + 8\hat{j} - 3\hat{k}$; $B \equiv \hat{j} + 2\hat{k}$

ASSIGNMENT

Ans. 7 $\overrightarrow{NP} = \frac{3}{4}\overrightarrow{AB} - \frac{1}{2}\overrightarrow{AF}$

Ans 10 $\vec{r}\cdot\left(\vec{n}_1 - \lambda\vec{n}_2\right) = d_1 - \lambda d_2$ where $\lambda = \frac{\left[\vec{n}_1\ \vec{n}_3\ \vec{n}_4\right]}{\left[\vec{n}_2\ \vec{n}_3\ \vec{n}_4\right]}$

Ans 28 $\frac{L}{2}$