

Number System

331
6 is divided by 7
Remainder

1.

What will be remainder,
when 127^{2271} is divided by 4?

$$\begin{aligned} 6^1 &= 6 \\ 6^2 &= 36 \\ \frac{36}{7} &= 1 \text{ remainder } 1 \\ 6^3 &= 216 \\ \frac{216}{7} &= 6 \text{ remainder } 6 \\ 6^4 &= 1296 \\ \frac{1296}{7} &= 185 \text{ remainder } 1 \\ 6^5 &= 7776 \\ \frac{7776}{7} &= 1111 \text{ remainder } 1 \\ 6^6 &= 46656 \\ \frac{46656}{7} &= 6665 \text{ remainder } 1 \end{aligned}$$

$$\frac{7^1}{4} = 3$$

$$\frac{7^2}{4} = 1$$

2701 is divided by 7

$$\begin{aligned} 4^1 &= 4 \\ 4^2 &= 2 \end{aligned}$$

$$\frac{4^3}{7} = \frac{64}{7} = 1$$

$$\frac{4^4}{7} = \frac{256}{7} = 4$$

49 343

2. What will be remainder when
 $2311 \times 3431 \times \overset{813}{\cancel{5691}}$ is divided by 7?

0

3. What will be remainder when 2311
× 435 × 596 is divided by 13?

$$\begin{array}{r} \cancel{4} \quad \cancel{1} \quad \cancel{7} \quad \cancel{2} \quad \textcircled{3} \\ \cancel{356} \times \cancel{221} \times \cancel{436} \times \cancel{519} \\ \hline \end{array} \quad = 1 \text{ A}$$

(1)

If any no n is divisible P
Then for all natural no k n^k is divisible by P .

4. When $(77)^{220}$ is divided by 7 then what will be remainder?

$$(94)^2$$

$$\frac{77}{7}$$

$$(77)^3 = \frac{77 \times 77 \times 77}{7}$$

$$\frac{(77)^2}{7} = \frac{77 \times 77}{7}$$

5. When 4^{2207} is divided by 6, then what will be remainder?

$$\frac{4^1}{6} = 4$$
$$\frac{4^2}{6} = \frac{16}{6} = 4$$
$$\frac{4^3}{6} = \frac{64}{6} = 4$$

6. If a number is divided by 7 it leaves remainder 5 but when it is divided by 5 it leaves remainder 3. What will be remainder when the same number is divided by 35?



Number = divisor \times quotient + remainder

$$\begin{aligned} \text{No.} &= 7x + 5 \\ &= 5y + 3 \end{aligned}$$

$$7x + 5 = 5y + 3$$

$$y = \frac{7x + 2}{5}$$

$$\begin{aligned} \text{least no} &= 7x + 5 \\ &= 7 \times 4 + 5 = 33 \end{aligned}$$

$$x = 4, y = 6$$

7. If a number is divided by 5 and 7 successively it leaves remainder 3 and 2 respectively. What will be remainder when the same number is divided by

Note:- ^{35?} Any no with divisor 7 and remainder 2 will be of the form $7k+2$ $35k+13$

When this no is divided by 5 leaves remainder 3
 $\frac{1}{2}$ 10-1 - Number = $5(7k+2) + 3 = 35k + 10 + 3 = 35k + 13$

\downarrow

(48), (83)

48

$$\begin{array}{r} 5 \overline{) 48} \quad (9 \\ \underline{45} \\ 3 \end{array}$$

$$\begin{array}{r} 7 \overline{) 91} \\ \underline{7} \\ 20 \\ \underline{14} \\ 6 \end{array}$$

$$\begin{array}{r} 5 \overline{) 83} \quad (16 \\ \underline{80} \\ 3 \end{array}$$

$$\begin{array}{r} 7 \overline{) 16} \quad (2 \\ \underline{14} \\ 2 \end{array}$$

$\frac{(10)}{7} =$

8. How many times digit 3 will appear up to 1000?

Unit place \rightarrow 100 times

001	30 - 39 \rightarrow 10 times
002	130 - 139 \rightarrow 10 times
003	230 - 239 \rightarrow 10 times
004	!
005	!
006	930 - 939 \rightarrow 10 times
007	<u>100 times</u>
008	
009	300 - 399 \rightarrow 100 times
010	

9. How many times digit 7 will appear between 550 to 800?

50 Num? → 550 - 599
 $\downarrow \downarrow \downarrow$
 $\times 10 5$

100 Num → 600 - 699
 $\downarrow \downarrow \downarrow$
 $\times 10 10$

100 Num → 700 - 799
 $\downarrow \downarrow \downarrow$
 $100 10 10$

Total no = $120 + 20 + 15$
 $= 155$ Ans

How many times 9 will appear from 200 to 500?

200 - 299
 $\downarrow \downarrow \downarrow$
 $100 10 10$

300 - 399
 $\downarrow \downarrow \downarrow$
 $\times 10 10$

400 - 499
 $\downarrow \downarrow \downarrow$
 $\times 10 10$

200 - 299 → 120

300 - 399 → 20

400 - 499 → 20

160 Ans

$$\begin{array}{r}
 2 \overline{) 360} \\
 \underline{720} \\
 2 \overline{) 180} \\
 \underline{360} \\
 2 \overline{) 90} \\
 \underline{180} \\
 2 \overline{) 45} \\
 \underline{90} \\
 3 \overline{) 15} \\
 \underline{45} \\
 3
 \end{array}$$

$$4 \times 3 \times 2 = 24$$

10. What will be the total no. of divisors of 360? $360 = 2^3 \times 3^2 \times 5^1 =$

N is any natural no. such that

$$N = a^p b^q c^r \dots$$

where a, b, c are prime no.
 p, q, r are natural no.

Then

- (i) The total no. of divisors of $N = (p+1)(q+1)(r+1) \dots$
- (ii) The product of all divisors of $N = \frac{1}{2}(p+1)(q+1)(r+1) \dots N$
- (iii) The sum of all divisors = $\frac{a^{p+1}-1}{a-1} \times \frac{b^{q+1}-1}{b-1} \times \frac{c^{r+1}-1}{c-1}$

what will be the total no. of divisors 180.

$$180 = 2^2 \times 3^2 \times 5^1$$

$$\text{total no. of divisors} = (2+1)(2+1)(1+1) = 3 \times 3 \times 2 = 18$$

- (1), 2, 3, 4, 5, 6, 9, 10, 12, 15, 18, 20, 30, 36, 45, 60
 90, 180

$$\begin{aligned}
 \text{Product of all divisors of } 180 &= N^{\frac{1}{2}(p+1)(q+1)(r+1)} \\
 &= (180)^9 \text{ Ans}
 \end{aligned}$$

proper divisors :- do not include 1 and no. itself.

$$\begin{aligned}
 \text{no. of proper divisors of } 180 &= 18 - 2 = 16 \\
 \text{product of proper divisors of } 180 &= \frac{(180)^9}{180 \times 1} = (180)^8 \text{ Ans}
 \end{aligned}$$

11. Find the sum of all divisors of 240?

$$240 = 2^4 \times 3^1 \times 5^1$$

$$\begin{aligned} \text{sum of all divisors} &= \frac{2^5 - 1}{2 - 1} \times \frac{3^2 - 1}{3 - 1} \times \frac{5^2 - 1}{5 - 1} \\ &= 31 \times 4 \times 6 = \underline{\underline{744}} \end{aligned}$$

• eg To find sum of proper divisors of 240

$$= 744 - (1 + 240)$$

$$= \underline{\underline{503}}$$

12. Find the Sum of all even Proper divisors

of 720. $= 2^4 \times 3^2 \times 5$

Sum of all even divisors = Sum of all divisors - Sum of all odd divisors

$$= \left(\frac{2^5-1}{2-1} \times \frac{3^3-1}{3-1} \times \frac{5^2-1}{5-1} \right) - \left(\frac{3^3-1}{3-1} \times \frac{5^2-1}{5-1} \right)$$

$$= 31 \times 13 \times 6 - 13 \times 6$$

$$= (31-1) \times 13 \times 6 = 30 \times 78 = \underline{\underline{2340}} \text{ Ans}$$

The
13. Find the sum of all odd proper divisors of 540.

$$540 = 2^2 \times 3^3 \times 5^1$$

$$\text{sum of all odd divisors} = \frac{3^4 - 1}{3 - 1} \times \frac{5^2 - 1}{5 - 1}$$

$$= 40 \times 6$$

$$= 240$$

$$\text{But sum of all odd proper divisors} = 240 - 1$$

$$= 239$$

14. Which of the following will divide the number

$$721^{2271} + 127^{2271} \text{ is divisible } 721+127 = \textcircled{848}$$

- (a) 604 ~~(b) 848~~ (c) 488 (d) 844

$a^n + b^n$ is divisible by $a+b$ if n is odd no.

$a^n + b^n + c^n$ is divisible by $a+b+c$ if n is odd no

$a^n - b^n$ is divisible by $a-b$ if n is odd no.

$a^n - b^n$ is divisible by $a-b$ and $a+b$ if n is even.

15. What will be remainder, when

$17^{71} \times 11^{71} \times 13^{71} \times 9^{71}$ will be divided by 25?

n is odd

is divisible by

$$17+11+13+9 = 50 \downarrow \text{divisible by } 50$$

Ⓐ 0

\therefore also " " 25

16. What will be remainder when 13^{71} is divided by 12?

Trick:- If n is any prime no. then

$\frac{n^k}{n-1}$ leaves always remainder 1.

17. How many numbers are there between 1 to 500 which are divisible by 3 or 4?

$$\frac{500}{3} = 166 \rightarrow 3, 6, 9, 12, 15, 18, 21, 24, \dots, 36, \dots$$

$$\frac{500}{4} = 125 \dots 4, 8, 12, 16, 20, 24, \dots, 36, \dots$$

$$\frac{500}{12} = 41 \text{ Total no which is either}$$

divisible by 3 or 4

$$= 166 + 125 - 41$$

$$= 291 - 41 = \underline{\underline{250}} \text{ Ans}$$



18. If 127 coins are to be kept in some small box each of Rs.1 in such a way that requirement of any amount between Rs. 1 to 127 can be given without further rearrangement. What will be the minimum number of box required?

$$\frac{(1000)}{7} = 4 \quad \frac{(1000)}{7} = 3 \quad \frac{(100)}{7} = 2$$

$$\frac{(100)}{7} = 150$$